Current Trends in Dynamical Systems and the Mathematical Legacy of Rufus Bowen

July 30- August 4, 2017
University of British Columbia, Vancouver

Conference Schedule
### Program at a Glance

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<tr>
<th>Time</th>
<th>Sunday 30</th>
<th>Monday 31</th>
<th>Tuesday 1</th>
<th>Wednesday 2</th>
<th>Thursday 3</th>
<th>Friday 4</th>
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<tbody>
<tr>
<td>8:00</td>
<td>Registration and Check in</td>
<td>ESB Atrium</td>
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<td>ESB Atrium 2207 Main Mall</td>
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<tr>
<td>8:45</td>
<td>Welcome From PIMS and Orgs</td>
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<tr>
<td>9:00</td>
<td>Steve Smale</td>
<td>Caroline Series</td>
<td>Yakov Pesin</td>
<td>Benjamin Weiss</td>
<td>Mike Hochman</td>
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<td>10:00</td>
<td>Coffee Break</td>
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<tr>
<td>10:30</td>
<td>Omri Sarig</td>
<td>Poster session 1</td>
<td>Ian Putnam</td>
<td>Poster session 2</td>
<td>Ronnie Pavlov</td>
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<tr>
<td>11:30</td>
<td>Jérôme Buzzi</td>
<td>Alex Blumenthal</td>
<td>Ursula Hamenstadt</td>
<td>David Ruelle</td>
<td>Vaughn Climenhaga</td>
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<tr>
<td>12:30</td>
<td>Arrivals and check-in to suites at UBC: *Ponderosa Commons, *Totem Park, *Carey Centre Residence</td>
<td>Lunch, Hosted</td>
<td>Lunch, Hosted</td>
<td>Lunch, Remembrance Lunch @ the UBC Alumni Centre 6163 University Blvd</td>
<td>Lunch, Hosted</td>
<td>Lunch, Hosted</td>
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<tr>
<td>14:00</td>
<td>Anatole Katok</td>
<td>Masato Tsujii</td>
<td>Jean-René Chazottes</td>
<td>Mark Pollicott</td>
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<td>15:00</td>
<td>Coffee Break</td>
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<tr>
<td>15:30</td>
<td>Rufus Notebook</td>
<td>Lewis Bowen</td>
<td>Lai- Sang Young</td>
<td>Wrap up</td>
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<tr>
<td>16:30</td>
<td>Problem session 1</td>
<td>Free afternoon+ optional activities</td>
<td>Problem session 2</td>
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<tr>
<td>17:30</td>
<td>Meet &amp; Greet @ The Nest, Rooftop Garden 4th floor 6133 University Blvd (prior RSVP required)</td>
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*There will be photography throughout this event. PIMS' event photography is used across a variety of our communications platforms including web, print and electronic promotional materials. If, for any reason, you wish not to have your photo taken or used in this manner, please contact the event organizers.*
Getting Started

Get connected: Select the “ubcvisitor” wireless network on your wireless device. Open up a web browser, and you will be directed to the login page. You can also log in through “eduroam” if this is available through your university.

FAQs

Q: Where do I check in on Monday Morning?

Check-in and package pick up can be done in the Earth Sciences Building (ESB) Atrium.

Q: Where are the sessions?

- All plenary sessions will be in the Earth Sciences Building (ESB) Room 1013.
- Problem sessions will be in Neville Scarfe and ESB 1013 and will be announced prior to the session.
- You will find a copy of the building floor plan on page 3 and a campus map at the end of the program.

Q: Will the program change? Program changes and updates will be announced at each session.

Q: When should I wear my badge? Please wear your name badges at all times on site so that PIMS Staff recognize you as a guest.

Q: Where can I go for help on site? If you need assistance or have a question, please feel free to talk to us at the registration desk.

Q: Where can I get refreshments and meals? For snacks or quick meals, please view the list of UBC eateries online at http://www.food.ubc.ca/feed-me/. Coffee breaks and lunches are provided each day of the workshop.

Q: Where can I get a cab to pick me up from the Venue? You can call Yellow Cab (604-681-1111) and request to be picked up at the intersection of West Mall and Bio. Sciences Road. Use the South West entrance of ESB, and walk straight down to the intersection.

Q: How can I get around?

- UBC Map link: Here
- Public Transit: Feel free to search and plan your public transport rides by visiting http://www.translink.ca/, where directions, ticket costs and bus schedules are indicated.
- Parking at UBC: http://www.parking.ubc.ca/visitor.html

Q: What emergency numbers should I know?

- Campus security (604-822-2222);
- General Emergencies (911);
- UBC hospital (604-822-7121).
Conference Room Guide:

Main Sessions:
Earth Sciences Building, 2207 Main Mall
Room ESB 1013

General Travel Directions:

Airport to UBC: Easiest by taxi (25min, around $30). By public transport, take the Canada Line (rail) to Broadway-City Hall station. From Broadway-City Hall station, cross Broadway and Cambie streets to get to the #99 UBC bus stop in front of London Drugs. Tickets (valid for the whole journey to UBC) can be purchased from the machine in the airport station. Cost: approximately $6. Journey time: Circa more than 1 hour

UBC Bus Loop/ Gage to Earth Science Building (ESB) 2207 Main Mall: A quick 10min walk. See UBC map. Head west past the student union building, cross East Mall and get onto Main Mall. Turn left (South) on Main Mall and Earth Science Building will be on your right after a few minutes. It is a large new building, and is on Main Mall directly across from the Beatty Biodiversity Centre and prominent blue whale skeleton.
Sunday July 30, 2017

5:30pm - 7:00pm  Optional Meet and Greet:
Light refreshments and nibbles served
AMS Nest
6133 University Blvd, Vancouver, BC V6T 1Z1

Monday July 31, 2017

8:00am- 8:45am  Registration and Check in (ESB Atrium)
8:45am - 9:00am  Opening remarks and announcements from PIMS and the Event Organizers.
9:00am - 10:00am  Steve Smale, University of Carlifornia at Berkeley
Problem of introducing dynamical systems into genome biology
10:00am – 10:30am  Coffee break
10:30am - 11:30am  Omri Sarig, Weizmann Institute of Science
Equilibrium states and the ergodic theory of positive entropy surface diffeomorphisms
11:30am - 12:30pm  Jérôme Buzzi, Université Paris-Sud Orsay
Bowen factors of Markov shifts and surface diffeomorphisms
12:30pm - 2:00pm  Hosted Lunch: ESB Atrium; Please show/have your name tag at this break.
2:00pm – 3:00pm  Anatole Katok, Pennsylvania State University
Entropy and lyapunov exponents: rigidity vs. flexibility
3:00pm - 3:30pm  Coffee break
3:30pm - 4:30pm  Rufus Bowen Notebook

Tuesday August 1, 2017

9:00am - 10:00am  Caroline Series, University of Warwick
The Bowen-Series coding - A survey
10:00am – 10:30am  Coffee Break
10:30am – 11:30am  Poster Session 1: ESB Atrium
11:30am – 12:30am  Alex Blumenthal, University of Maryland
Lyapunov exponents for small random perturbations of predominantly hyperbolic two dimensional volume-preserving diffeomorphisms, including the Standard Map
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<tr>
<td>2:00pm – 3:00pm</td>
<td><strong>Masato Tsujii, Kyushu University</strong></td>
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<td><em>Exponential decay of correlations for volume preserving Anosov flows in dimension 3</em></td>
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<td>3:00pm – 3:30pm</td>
<td>Coffee Break</td>
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<td>3:30pm – 4:30pm</td>
<td><strong>Lewis Bowen, University of Texas</strong></td>
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<td><em>Entropy theory in the nonamenable setting</em></td>
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<tr>
<td>4:30pm – 5:30pm</td>
<td>Problem session 1 (Neville Scarfe Building) Session titles and room numbers TBC</td>
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**Wednesday August 2, 2017**

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<td>9:00am - 10:00am</td>
<td><strong>Yakov Pesin, Pennsylvania State University</strong></td>
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<td><em>A geometric approach for constructing SRB measures in hyperbolic dynamics</em></td>
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<tr>
<td>10:00am – 10:30am</td>
<td>Coffee break</td>
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<tr>
<td>10:30am – 11:30am</td>
<td><strong>Ian F. Putnam, University of Victoria</strong></td>
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<td><em>A homology theory for Smale spaces</em></td>
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<td>11:30am – 12:30pm</td>
<td><strong>Ursula Hamenstadt, Universität Bonn</strong></td>
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<td><em>Amenable actions, Lyapunov exponents and an idea of Bowen</em></td>
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<tr>
<td>12:30pm – 3:00pm</td>
<td>Rufus Bowen Remembrance Lunch:</td>
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<td>UBC Alumni Centre:</td>
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<td><strong>6163 University Blvd</strong></td>
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<td><em>Please wear your name tag for this event.</em></td>
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<tr>
<td>3:00pm</td>
<td>Free afternoon/optional activities including a short hike (See page 19 - 20 of program)</td>
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**Thursday August 3, 2017**

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<tr>
<th>Time</th>
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<tr>
<td>9:00am – 10:00am</td>
<td><strong>Benjamin Weiss, Hebrew University of Jerusalem</strong></td>
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<td><em>Smooth models for ergodic systems</em></td>
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<tr>
<td>10:00am – 10:30am</td>
<td>Coffee break</td>
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<tr>
<td>10:30am – 11:30am</td>
<td>Poster session 2: ESB Atrium</td>
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<tr>
<td>11:30am – 12:30pm</td>
<td><strong>David Ruelle, IHES</strong></td>
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A theory of turbulence based on non-equilibrium statistical mechanics

12:30pm - 2:00pm  Hosted Lunch: ESB Atrium; please show/have your name tag at this break

2:00pm- 3:00pm  Jean-René Chazottes, CNRS

Gibbs measures and equilibrium states at low temperature

3:00pm - 3:30pm  Coffee break

3:30pm - 4:30pm  Lai-sang Young, Courant Institute & NYU

Natural invariant measures

4:30pm - 5:30pm  Problem session 2 (Neville Scarfe Building) Session titles TBC

Friday August 4, 2017

9:00am – 10:00am  Mike Hochman, Hebrew University of Jerusalem

Borel isomorphism of systems with some hyperbolic structure

10:00am – 10:30am  Coffee break

10:30am – 11:30am  Ronnie Pavlov, University of Denver

Non-uniform specification properties for subshifts

11:30am – 12:30pm  Vaughn Climenhaga, University of Houston

Unique equilibrium states for geodesic flows in nonpositive curvature

12:30pm – 2:00pm  Hosted Lunch: ESB Atrium; please show your name tag at this break.

2:00pm – 3:00pm  Mark Pollicott, Warwick University

Bowen’s dimension formula and rigorous estimates

3:00pm – 3:30pm  Coffee break

3:30pm – 4:30pm  Conference Wrap-Up
Speaker Titles and Abstracts

1. **Alex Blumenthal, University of Maryland**

   *Lyapunov exponents for small random perturbations of predominantly hyperbolic two dimensional volume-preserving diffeomorphisms, including the Standard Map*

   An outstanding problem in smooth ergodic theory is the estimation from below of Lyapunov exponents for maps which exhibit hyperbolicity on a large but non-invariant subset of phase space. It is notoriously difficult to show that Lyapunov exponents actually reflect the predominant hyperbolicity in the system, due to cancellations caused by the “switching” of stable and unstable directions in those parts of phase space where hyperbolicity is violated. In this talk I will discuss the inherent difficulties of the above problem, and will discuss recent results when small IID random perturbations are introduced at every time-step. In this case, we are able to show with relative ease that for a large class of volume-preserving predominantly hyperbolic systems in two dimensions, the top Lyapunov exponent actually reflects the predominant hyperbolicity in the system. Our results extend to the well-studied Chirikov Standard Map at large coupling. This work is joint with Lai-Sang Young and Jinxin Xue.

2. **Lewis Bowen, University of Texas**

   *Entropy theory in the nonamenable setting*

   I’ll explain a recent extension of Kolmogorov-Sinai entropy from actions of Z to actions by much larger groups such as free groups and SL(n,Z). Applications include a classification of Bernoulli shifts over these groups up to measure-conjugacy. I’ll also talk about more recent developments such as applications to algebraic dynamics or generalizations of Ornstein theory if time permits.

3. **Jérôme Buzzi, Université Paris-Sud Orsay**

   *Bowen factors of Markov shifts and surface diffeomorphisms*

   Classical theorems of Sinai and Bowen show that diffeomorphisms admit very simple symbolic dynamics. More precisely, they are factors of shifts of finite type. These factor maps are usually not injective but this failure has a combinatorial description, noticed by Bowen. This ”Bowen property” was considered by several authors, including Manning (for zeta functions) and Fried (finitely presented systems).

   It turns out that the Bowen property extends to the non-uniform case, namely to the symbolic dynamics built by Sarig for surface diffeomorphisms. I will explain the role of this property in the analysis of their ergodic theory (joint with Boyle) and especially of the number of measures maximizing the entropy (joint with Crovisier and Sarig) as well as lower bounds on their number of periodic points.

4. **Jean-Rene Chazottes, CNRS**

   *Gibbs measures and equilibrium states at low temperature*

   We will be interested in what happens to a Gibbs measure when temperature is ”low” or goes to 0. Our main motivation is to understand how ”quasicrystals” can arise. We will deal with lattice models: at each site of the d-dimensional cubic lattice, one can have a finite number of possible states, e.g., 0 or 1. We will report on two types of behaviours: 1. the existence (or not) of the limit when temperature goes to 0, and what is the nature of the limiting measure when it exists; 2. freezing phase transitions, that is, the fact that for some non-zero critical temperature, the support of the Gibbs measure shrinks dramatically and one gets an equilibrium...
state living on a substitution subshift for all temperatures below the critical one. In the latter case, nothing is known for dimension greater than or equal to two.

5. Vaughn Climenhaga, University of Houston

Unique equilibrium states for geodesic flows in nonpositive curvature

The geodesic flow on a negatively curved manifold is one of the classical examples of a uniformly hyperbolic (transitive Anosov) system; in particular, results of Bowen, Ruelle, and Sinai show that it has a unique measure of maximal entropy, and more generally, unique equilibrium states for Holder continuous potentials. When curvature is only assumed to be nonpositive, the geodesic flow becomes nonuniformly hyperbolic and much less is known. For a rank 1 manifold of nonpositive curvature, Knieper showed uniqueness of the measure of maximal entropy, but his methods do not generalize to equilibrium states for nonzero potentials. I will discuss joint work with Keith Burns, Todd Fisher, and Daniel J. Thompson, in which we use a nonuniform version of Bowen’s specification property to establish existence and uniqueness of equilibrium states for a class of nonzero potential functions; this class includes scalar multiples of the geometric potential for an interval of parameter values, and includes an open and dense set of Holder continuous potentials in the specific case when the manifold has dimension two and the metric is analytic.

6. Ursula Hamenstadt, Universität Bonn

Amenable actions, Lyapunov exponents and an idea of Bowen

Given a random walk on a group $G$ and a homomorphism $B$ of $G$ into $GL(n,R)$, simplicity of the Lyapunov spectrum for the corresponding cocycle over the Markov shift defined by the random walk is well known to be a consequence of algebraic properties of $B$. We discuss an extension of this simplicity result to cocycles over a class of flows with some hyperbolicity properties and with respect to a sufficiently nice probability measure. This class of flows includes all geodesic flows on finite volume negatively curved manifolds. We also give some applications. The proof uses an idea of Bowen and does not rely on tools from random walks.

7. Mike Hochman, Hebrew University of Jerusalem

Borel isomorphism of systems with some hyperbolic structure

Systems with some hyperbolic structure have long been known to be essentially classified (up to isomorphism of large subsets of the systems) by entropy and periodic points. Such results can take several forms e.g. a continuous equivariant bijection on a dense $G_delta$ set, or a measurable one a set supporting measures of large entropy, etc. All this raises the question of whether one can get a full isomorphism of the Borel structure. It turns out that the (affirmative) answer involves the study of the irregular points in the system. In the talk I will try to explain this problem in more detail, and some ingredients in its solution.

8. Anatole Katok, Pennsylvania State University

Entropy and Lyapunov exponents: rigidity vs. flexibility

The short but very influential and highly quoted Bowen’s 1971 Transactions paper “Entropy for group endomorphisms and homogeneous spaces” can be viewed as the starting point for the study of two complementary phenomena: rigidity and flexibility for conservative (volume-preserving) systems. Bowen showed that for dynamical systems of algebraic origin, i.e. homogeneous and affine maps and flows on homogeneous spaces, the values of metric entropy with respect to Haar measure and topological entropy are always the same, and the range of their values (as well as the values of Lyapunov exponents) is limited.
Notice that the only situation beyond the algebraic ones where either invariant can be calculated precisely is discrete time Anosov systems on infranilmanifolds, including tori, that are topologically conjugate to the algebraic models. There is a number of situations where the values of those invariants determine algebraic systems within a large class of systems. Those are prototype rigidity results. On the other hand, beyond the algebraic case the flexibility paradigm should hold: Under properly understood general restrictions within a fixed class of smooth dynamical systems quantitative dynamical invariants take arbitrary values.

Most known constructions are perturbative and hence at best would allow to cover a small neighborhood of the values allowed by the model, or more often, not even that, since homogeneous systems are often “extremal”. So establishing flexibility calls for non-perturbative or large perturbation constructions in large families to cover possible values of invariants.

Work on flexibility is still in its infancy and in many situations the proper “general restrictions” are not fully understood. In this talk I will discuss general conjectures and first two results in their direction.

9. Ronnie Pavlov, University of Denver

Non-uniform specification properties for subshifts

The celebrated specification property of Bowen implies many useful properties for an expansive topological dynamical system, among them intrinsic ergodicity, i.e. uniqueness of the measure of maximal entropy. In the setting of symbolic dynamics, this property is equivalent to the existence of a constant N such that any two n-letter words v,w in the language can be combined into a new word in the language given a gap between them of length at least N.

There are several natural weakenings of specification in the symbolic setting; for instance one can allow a gap which grows as a function f(n), or one can allow a small number g(n) of “edits” to the letters of v,w instead of leaving a gap; we call these properties non-uniform gap specification and non-uniform mistake specification. I will summarize some results about these properties, focusing mostly on thresholds for f(n) and g(n) which are known to either guarantee intrinsic ergodicity or allow for multiple measures of maximal entropy.

10. Yakov Pesin, Pennsylvania State University

A geometric approach for constructing SRB measures in hyperbolic dynamics

In dissipative dynamics hyperbolic SRB measures form a class of natural and physically meaningful measures supported on attractors and they have a rich collection of ergodic properties (up to the Bernoulli property). They were introduced and studied in seminal works of Sinai, Ruelle, and Bowen in the case when an attractor is uniformly hyperbolic, in particular, when the map is Anosov. The construction of SRB measures in these works uses finite Markov partitions that allow one to represent the system as a subshift of finite type. In more general situations one can make use of representing the attractor by a countable state Markov shift or a Young tower. I will outline a unified approach for constructing SRB measures in hyperbolic dynamics which is pure geometrical in its nature and which does not use any symbolic model of the system. This approach can be used to construct SRB measures for a broad class of non-uniformly hyperbolic “chaotic” attractors where building a symbolic model of the system may not be possible.

11. Mark Pollicott, Warwick University

Bowen’s dimension formula and rigorous estimates

In Bowen’s 1979 paper (published posthumously) he introduced a now famous formula which is used to give the dimension of a conformal repeller implicitly in terms of the pressure function. This is his 4th most cited publication on MathSciNet. In 1982,
Ruelle-related this to the dynamical zeta function. In this talk we will describe how these ideas can be further developed to give a method which accurately and rigorously estimates the dimension in some specific interesting cases.

12. Ian F. Putnam, University of Victoria
   *A homology theory for Smale spaces*

   Smale spaces were introduced by David Ruelle to provide an axiomatic framework for the dynamics on the basic sets of Smale’s Axiom A systems. Anthony Manning proved that the Artin-Mazur zeta function for such a system is rational, leading Bowen to conjecture the existence of a homology theory for such systems which gave a Lefschetz-type formula for the number of periodic points of the system as the trace of a linear map. Such a theory was given by Krieger for shifts of finite type.

   (Another was given by Bowen and Franks in this same case.) In this talk, we show how Krieger’s theory can be extended to all Smale spaces. The key technical ingredient is the existence of a better type of Markov partition.

13. David Ruelle, IHES
   *A theory of turbulence based on non-equilibrium statistical mechanics*

   Abstract. In earlier work we have studied the turbulent flow exponents $\zeta_p$, where $\langle |\Delta v|^p \rangle \sim \ell^{\zeta_p}$ and $\Delta v$ is the contribution to the fluid velocity at small scale $\ell$. Using ideas of non-equilibrium statistical mechanics we have found

   $$\zeta_p = \frac{p}{3} - \frac{1}{\ln \kappa} \ln \Gamma\left(\frac{p}{3} + 1\right)$$

   where $1/\ln \kappa$ is experimentally $\approx 0.32 \pm 0.01$. The purpose of the present note is to propose a somewhat more physical derivation of the formula for $\zeta_p$. We also present an estimate $\approx 100$ for the Reynolds number at the onset of turbulence.

14. Omri Sarig, Weizmann Institute of Science
   *Equilibrium states and the ergodic theory of positive entropy surface diffeomorphisms*

   I will revisit Bowen’s famous 1975 monograph “Equilibrium states and the ergodic theory of Anosov diffeomorphisms”, and discuss the parts of this text which we can now extend to general $C^{1+\epsilon}$ surface diffeomorphisms with positive topological entropy.

15. Caroline Series, University of Warwick
   *The Bowen-Series coding - A survey*

   We will give a survey of the Bowen-Series coding, including its genesis, extensions, and a variety of applications old and new.

16. Steve Smale, University of California at Berkeley
   *Problem of introducing dynamical systems into genome biology*

   We address 2 problems of the title. 1) formalizing a dynamical system for gene expression levels 2) cellular dynamics where the states of the cells are equilibrium protein distributions. (so that the cell itself can change in time). The relations between 1) and 2) will be discussed.
17. Masato Tsujii, Kyushu University

Exponential decay of correlations for volume preserving Anosov flows in dimension 3

We present a result that generic volume-preserving Anosov flows on 3-dimensional manifolds are exponentially mixing. The proof is based on analysis of local geometric structure of the strong stable and unstable foliation. In the talk, I would like to discuss a generalization of the result to higher dimensional cases.

18. Benjamin Weiss, Hebrew University of Jerusalem

Smooth models for ergodic systems

Rufus Bowen made many fundamental contributions to our understanding of the ergodic theoretic properties of invariant measures for smooth mappings. I will describe some of the recent progress that has been made (jointly with Matt Foreman) on the basic question: How large is the class of ergodic transformations that have as a model a smooth mapping of a compact manifold preserving a volume element?

19. Lai-sang Young, Courant Institute & NYU

Natural invariant measures

The theory of SRB measures for Axiom A attractors has been the motivation for much of the development in the subject in the last 40 years. One of Rufus’ legacies is his contribution to this theory. In this talk I would like to review some of the progress that has been made since, including extensions to nonuniformly hyperbolic systems, to random dynamical systems, to infinite dimensional systems (so the theory applies to some PDEs as well as ODEs), and to a range of other settings relevant to applications. Both theory and examples will be discussed. The exposition will be somewhat personal in that it will be biased toward my own work and the work of my collaborators.

Poster Titles and Abstracts

1. Jason Atnip, University of North Texas

Dimension of Julia Sets of Certain Classes of Non-Autonomous Meromorphic Functions

In this poster we study two classes of meromorphic functions of finite order previously studied by Mayer in 2007 and by Kotus and Urbanski in 2008. In particular, under mild assumptions on the behavior of the functions near poles, we are able to estimate a lower bound for the Hausdorff dimension of the Julia set and an upper bound for the Hausdorff dimension of the set of escaping points for non-autonomous perturbations of functions from these two classes. We accomplish this by constructing non-autonomous iterated function systems, as described by Rempe and Urbanski, which sit inside of the aforementioned non-autonomous Julia sets.

2. Raimundo Briceño, Tel Aviv University

New techniques for pressure representation in $Z^d$ shift spaces

Building upon previous work from several authors, given a $Z^d$ shift space and a shift-invariant potential, we present new sufficient conditions for efficient approximation of pressure and, in particular, topological entropy. First, we explain some previous results that were based on a measure-theoretic property of Gibbs measures known as strong spatial mixing and a combinatorial analogue introduced by the author. Next, we proceed to define strictly more general pointwise versions of these two properties, explore some
of their implications, and show that they are sufficient to obtain efficient approximation algorithms for pressure. Similar to previous results, the techniques make use of a special representation theorem for pressure but, in contrast, the new conditions may apply in more general supports and when there are multiple equilibrium states. Finally, we exhibit some natural examples supporting this last claim. This is joint work with Stefan Adams, Brian Marcus and Ronnie Pavlov.

3. Meagan Carney, University of Houston

**Dynamical Borel Cantelli lemmas and rates of growth of Birkhoff sums of non-integrable observables on chaotic dynamical systems**

We consider implications of dynamical Borel-Cantelli lemmas for rates of growth of Birkhoff sums of non-integrable observables \( \varphi(x) = d(x, q)^{-k} \), \( k > 0 \), on ergodic dynamical systems \( (T, X, \mu) \) where \( \mu(X) = 1 \). In this presentation we discuss a few general results as well as some more concrete examples involving non-uniformly expanding maps, intermittent type maps as well as uniformly hyperbolic systems.

4. Nishant Chandgotia, Tel Aviv University

**Skew Products Over The Irrational Rotation, The Central Limit Theorem And RATs**

Let \( f \) be a step function on the circle with zero mean and rational discontinuities while \( \alpha \) is a quadratic irrational. The point-wise ergodic theorem tells us that the ergodic sum, \( f(x) + f(x+\alpha) + \ldots + f(x+(n-1)\alpha) \) is \( o(n) \) for almost every \( x \) but says nothing about its deviations from zero, that is, its discrepancy; the study of these deviations naturally draws us to the study of ergodic transformations on infinite measure spaces, viz., skew products over irrational rotations. In this poster we will discuss how the temporal statistics of the ergodic sums for \( x=0 \) can be studied via random affine transformations (RATs) leading to a central limit theorem and other fine properties like the visit times to a neighbourhood of \( 0 \) vis-à-vis bounded rational ergodicity.

5. Felipe Garcia-Ramos, Universidad Autonoma de San Luis Potosi

**Synchronization on Bounded density shifts and beta shifts.**

Beta shifts and bounded density shifts are similarly defined symbolic systems. We characterize the synchronization property in both families.

6. Mukta Garg, University of Delhi

**On dynamics of maps possessing the almost average shadowing property**

Pseudo-orbits are obtained during computer simulation of an orbit in a discrete dynamical system and a system possessing the shadowing property forces these simulated orbits to follow true orbits of the system. Pseudo-orbits play an important role in investigating properties of the system like detecting mixing and recurrent behaviors of the system which may not be evident by studying actual orbits. Situations occur where instead of an exact error bound in each step of a simulated orbit, one gets a small average error in long runs of the orbit which motivated us to define the notion of almost average shadowing property. Here we relate this notion with certain useful dynamical properties, for instance, variants of transitivity, different types of chaos (including topological entropy), proximality and some other types of shadowing properties. We also give some interesting examples and counterexamples to gloss the study done.
7. Ben Hayes, Vanderbilt University

Fuglede-Kadison determinants and sofic entropy

I will present on algebraic actions, which are actions of a countable, discrete group $G$ by automorphisms of a compact, abelian, metrizable group $X$. This is a probability measure-preserving action, giving $X$ the Haar probability measure. One is interested in classifying, or at least calculating invariants for, this class of probability measure-preserving actions. One common invariant to compute is the entropy of this action. I will present my results computing entropy for a large class of algebraic actions, when the group is sofic (the largest class of groups for which entropy is currently defined). The computation relates to operator algebraic invariants, namely the Fuglede-Kadison determinant.

8. Yan Mary He, University of Chicago

Basmajian-type Identities and Hausdorff Dimension of Limit Sets

We introduce Basmajian-type series identities on limit sets associated to familiar one-dimensional complex dynamical systems. We show that the series is absolutely convergent if and only if the Hausdorff dimension of the limit set is strictly less than one. In particular, this allows us to extend Basmajian’s identity to certain Schottky groups and to find the Hausdorff dimension one locus in the complement of the Mandelbrot set for quadratic polynomials.

9. Michihiro Hirayama, University of Tsukuba

On the ergodicity of hyperbolic Sinai-Ruelle-Bowen measures

For a hyperbolic Sinai-Ruelle-Bowen measure having intersections for almost every pair of the stable and unstable manifolds, the measure is ergodic if the unstable dimension is constant. When the dimension of the ambient manifold is less than or equal to three, the ergodicity follows without the condition of constancy of the unstable dimension. Further, several topological conditions, which guarantee that there exists at most one hyperbolic ergodic Sinai-Ruelle-Bowen measure, are obtained.

10. Daniel Ingebretson, University of Illinois at Chicago

Hausdorff dimension of Kuperberg minimal sets

The Seifert conjecture was answered negatively in 1993 by Kuperberg who constructed a smooth aperiodic flow on a three-manifold. This construction was later found to contain a minimal set with a complicated topology. This minimal set is embedded as a lamination by surfaces with a Cantor transversal of Lebesgue measure zero. In this talk we will discuss the pseudogroup dynamics on the transversal, the induced symbolic dynamics, and the Hausdorff dimension of the Cantor set.

11. Kieran Jarrett, University of Bath

Non-singular actions of the Heisenberg groups

We present a sequence of subsets of the discrete Heisenberg groups which, when taken as the range of sums in the time average, satisfy the ergodic theorem for any non-singular action of the group. We apply the ergodic theorem to study the critical dimension of such actions.
12. Naotaka Kajino, Kobe University

Weyl’s eigenvalue asymptotics for the Laplacian on circle packing limit sets of certain Kleinian groups

This poster presents the author’s recent results on the construction of a “canonical” Laplacian on circle lacking fractals invariant under the action of certain Kleinian groups and on the asymptotic behavior of its eigenvalues. In the simplest case of the Apollonian gasket, the Laplacian was constructed by Teplyaev (2004) as the unique one with respect to which the coordinate functions on the gasket are harmonic, and the author has recently discovered an explicit expression of it in terms of the circle packing structure of the gasket, which immediately extends to general circle packing fractals and defines (a candidate of) a “canonical” Laplacian on those fractals. When the circle packing fractal is the limit set of a maximal cusp group on the Maskit boundary of the quasifuchsian space, the author has also proved Weyl’s asymptotic formula for the eigenvalues of this Laplacian, which is of the same form as the circle-counting asymptotic formula by Oh and Shah (Invent. Math., 2012). The proof of this result heavily relies on ergodic-theoretic analysis of a Markov chain on a certain set of (equivalence classes of) Möbius transformations.

13. Sergey Komech, IITP RAS, Moscow, Russia

Deformation rate in dynamical systems

We study a relation between Kolmogorov entropy and boundary distortion rate. This phenomenon was mentioned for the first time in physics literature. There it is stated that the boundary of a set in the phase space of a dynamical system increases exponentially fast with coefficient that equals the entropy of the system corresponding to an invariant measure. But such a relation could be established only if there is some specific dependence between time and size of a set. First rigorous results were obtained by B. Gurevich for symbolic dynamical systems, precisely shifts of finite type. It turns out that the relation holds true for much wider class of dynamical systems: synchronized systems (which contain all sofic shifts), Anosov systems and some flows.

More precisely: In the case of symbolic dynamical systems we evaluate boundary distortion rate in terms of invariant measure, and establish that coefficient equals the entropy. In the case of smooth dynamical systems we evaluate boundary distortion rate in terms of Riemannian measure, which could be non-invariant, and establish that coefficient is equal to the sum of positive Lyapunov exponents of invariant measure (and hence, if invariant measure is Sinai-Ruelle-Bowen measure, to the entropy).

14. Robert Kozma, University of Illinois Chicago

Julia Sets Converging to Filled Quadratic Julia Sets

Previous results by Devaney et al. have shown that for the family of singularly perturbed quadratic maps $z^2 + \lambda/z^2$ the Julia sets converge to the unit disk as $\lambda \to 0$. We give a generalization of this result to maps of the family $F(z) = z^2 + c + \lambda/z^2$ where $c$ is the center of a hyperbolic component of the Mandelbrot set. Using symbolic dynamics, Markov partitions, and Cantor necklaces, we show that as $\lambda \to 0$, the Julia set of $F$ converges to the filled Julia set of $z^2 + c$ in the Hausdorff sense, thus coming arbitrarily close to containing an open set. This is surprising as it is well known that if a Julia set contains an open set it must be the entire complex plane.

15. Dominik Kwietniak, Federal University of CRio de Janeiro

Robust existence of nonhyperbolic ergodic measures with positive entropy and full support

We prove that for a large family of manifolds containing all manifolds supporting a transitive Anosov flow there is nonempty and open set of diffeomorphisms possessing a nonhyperbolic (with at least one zero Lyapunov exponent) ergodic measure with full
support and positive entropy. Several previous works established existence of nonhyperbolic measures in a similar setting and with some (but not all) of these properties. Our proof is based on a strengthening of the tools used by Boichi, Bonatti and Diaz in their recent construction of nonuniformly hyperbolic ergodic measures with full support. In order to show that the entropy of our measure is positive we use some single orbit techniques inspired by works of Kamae and Weiss. These are general results that hold beyond the setting of our paper.

16. Martha Łącka, Jagiellonian University in Krakow

*Feldman-Katok pseudometric and the GIKN construction of nonhyperbolic ergodic measures*

The *GIKN construction* was introduced by Gorodetski, Ilyashenko, Kleptsyn, and Nalsky. It gives a nonhyperbolic ergodic measure which is a weak$^*$ limit of a special sequence of measures supported on periodic orbits. This method provided examples of nonhyperbolic invariant measures in various settings. We prove that the result of the GIKN construction is always a loosely Kronecker measure in the sense of Ornstein, Rudolph, and Weiss (equivalently, standard measure in the sense of Katok). For a proof we introduce and study the Feldman-Katok pseudometric $\bar{F}_K$. The pseudodistance $\bar{F}_K$ is a topological counterpart of the *f-bar* metric for finite-state stationary stochastic processes introduced by Feldman and, independently, by Katok, later developed by Ornstein, Rudolph, and Weiss. We show that every measure given by the GIKN construction is the $\bar{F}_K$-limit of a sequence of periodic measures. We also prove that a measure which is the $\bar{F}_K$-limit of a sequence of ergodic measures is ergodic and its entropy is smaller or equal than the lower limit of entropies of measures in the sequence. Furthermore we demonstrate that $\bar{F}_K$-Cauchy sequence of periodic measures tends in the weak$^*$ topology either to a periodic measure or to a loosely Kronecker measure.

17. Bingbing Liang, Max-Planck Institute for Mathematics in Bonn

*Dynamical correspondences of $L^2$-Betti numbers*

We investigate dynamical analogues of the $L^2$-Betti numbers for modules over integral group ring of a discrete sofic group. As an application, we give a dynamical characterization of $L^2$-ck's dimension-flatness.

18. Marco Lopez, University of North Texas

*Dimension of Shrinking Targets from Non-Autonomous Systems*

The shrinking target problem refers to the study of the set of points in a metric space, whose orbits under a dynamical system hit infinitely often a ball with radius shrinking to zero. In our work we focus on establishing Bowen’s dimension formula for shrinking target sets in the context of non-autonomous iterated function systems on the unit interval. In special cases, such shrinking target sets arise in Diophantine approximation.

19. Jeovanny de Jesus Muentes Acevedo, University of Sao Paulo

*On the Continuity of the Topological Entropy of Non-autonomous Dynamical Systems*

Abstract Let $M$ be a compact Riemannian manifold. The set $F^r(M)$ consisting of sequences $(f_i)_{i \in \mathbb{Z}}$ of $C^r$-diffeomorphisms on $M$ can be endowed with the compact topology or with the strong topology. A notion of topological entropy is given for these sequences. I will prove this entropy is discontinuous at each sequence if we consider the compact topology on $F^r(M)$. On the other hand, if $r \geq 1$ and we consider the strong topology on $F^r(M)$, this entropy is a continuous map.
20. Chen-Chang Peng, Department of Applied Mathematics, National Chiayi University

Estimate Hausdorff dimension and box dimension for Mira maps near anti-integrable limits

First we introduce why we study Mira maps and the dynamics for Mira maps near anti-integrable limits. Near anti-integrable limits, the repellers for Mira maps coincide with the attractors of the related iterated function systems. We estimate the upper bounds and the lower bounds of the fractal dimension of the attractors for the iterated function systems. The invariant subsets for Mira maps near anti-integrable limits are hyperbolic repellers but they are not all conformal.

21. Vytnova Polina, University of Warwick

Towards a Kinematic Fast Dynamo

The kinematic fast dynamo equation describes evolution of the magnetic field of large astrophysical objects, such as planets and stars. It is a special case of the Navier Stokes equation in the real 3 dimensional space, with two parameters, a vector field and a diffusion coefficient. A divergent free vector field is called a fast dynamo, if the corresponding equation has a solution in which energy grows exponentially with time. The existence of this vector field is an open question. We will present a recent progress on this problem.

22. Gabriel Ponce, State University of Campinas, UNICAMP

Measure Rigidity and Disintegration: Time one map of flows.

The results presented in this poster are mainly related to the classification of ergodic measures which are invariant by the time-1 map of a measurable flow on a Lebesgue space. Let $\phi_t:X \rightarrow X$ be a continuous flow on a compact metric space. In this work we are able to understand the restriction for an ergodic invariant probability measure for the time one map $\phi_1:X \rightarrow X$ to be invariant and ergodic for the flow. This restriction is an interesting phenomena closely related to the behavior of a pathological foliation as firstly described by Milnor over a Katok’s example known as “Fubini’s Nightmar”. We also describe all ergodic measures for $\phi_1$ in terms of its Rohklin disintegration over the partition by orbits of the flow.

23. Luciana Salgado, UFBA/UFRJ

Dominated splittings for singular flows

We prove necessary conditions for an invariant splitting of the tangent bundle over a compact invariant set for a flow to be dominated. In particular, it is reduced to the requirements to obtain singular hyperbolicity. The flow known as ”Bowen Example” is used to show that the result is not true if we remove the domination hypothesis over singularities.

24. Vitalii Senin, TU Berlin

Pesin’s Formula for Isotropic Brownian Flows

Pesin's formula has never been established for dynamical systems with invariant measure, which is infinite. The problem is that in this case the notion of entropy becomes senseless. Isotropic Brownian flows is a specific class of random dynamical systems, which are defined on d-dimensional Euclidean space, and invariant in distribution with respect to rotations and translations. They are called Brownian, because under the action of an isotropic Brownian flow the image of a particular point behaves purely like a Brownian motion. The invariant measure of isotropic Brownian flows is the Lebesgue measure, which is, clearly, infinite. Nevertheless, we define entropy for such a class of flows using the notion of local entropy, which was introduced by Brin and Katok
in 1983. This idea makes sense because of translation invariance of isotropic Brownian flows. Then we study the analogue of Pesin’s formula for these flows using the defined entropy.

25. Mao Shinoda, Keio University

Properties of ergodic maximizing measures for dense continuous functions

Consider a continuous self-map of a compact metric space. The main purpose of ergodic optimization is to single out dynamically invariant Borel probability measures which maximize the integral of a given "performance" function. Jenkinson shows that for a generic continuous function there exists a unique maximizing measure. Moreover, Morris shows the unique maximizing measure of a generic continuous function is fully supported and has zero entropy, provided the self-map satisfies the specification property. However it is difficult to tell whether a given performance-function is generic. Indeed, no concrete example of a continuous function is known which is uniquely maximized by a fully supported measure. Hence it is natural to investigate properties which are not generic, yet hold for a reasonably large set of functions. In this poster we prove the existence of uncountably many ergodic maximizing measures for a dense subset of continuous functions. In the case of symbolic systems we can refine the property of the uncountably many ergodic maximizing measures.

26. Kathryn Spalding, Loughborough University

Growth of Values of Binary Quadratic Forms and Conway Rivers

We study the growth of the values of a binary quadratic form $Q$ on a binary planar tree as it was described by Conway. We show that the corresponding Lyapunov exponents $\Lambda_Q(x)$ as a function of path determined by $x \in \mathbb{R}P^1$ are twice the values of the corresponding exponents for the growth of Markov numbers, except for the paths corresponding to the Conway rivers, when $\Lambda_Q(x) = 0$. The relation with Galois results about continued fraction expansions for quadratic irrationals is explained and interpreted geometrically.

27. Adam Śpiewak, Warsaw University

A note on a Wiener-Wintner theorem for mean ergodic Markov amenable semigroups

Classical Wiener-Wintner ergodic theorem establishes almost sure convergence of a sequence $\frac{1}{N} \sum_{n=0}^{N-1} \lambda^n f(T^n x)$ for every $\lambda$ on the unit circle, where $T$ is an ergodic measure preserving transformation and $f : X \rightarrow \mathbb{C}$ is integrable. We prove a generalization of its topological version for semigroups of Markov operators on $C(X)$. We assume that $\{S_g : g \in G\}$ is a mean ergodic representation of a right amenable semitopological semigroup $G$ by linear Markov operators on $C(X)$, where $X$ is some compact Hausdorff space. The main result is necessary and sufficient conditions for mean ergodicity of a distorted semigroup $\{\chi(g)S_g : g \in G\}$, where $\chi$ is a semigroup character. Such conditions were obtained before under the additional assumption that $\{S_g : g \in G\}$ is uniquely ergodic. This is joint paper with Wojciech Bartoszek "A note on a Wiener-Wintner theorem for mean ergodic Markov amenable semigroups" (https://doi.org/10.1090/proc/13495).

28. Matteo Tanzi, Imperial College London

Heterogeneously Coupled Maps: hub dynamics and emergence across connectivity layers

We study expanding dynamical systems coupled on an heterogeneous network so that two systems interact if there is an edge of the network connecting them. In this setting, most systems make a small number of interactions, while those corresponding to the
highly connected nodes (hubs) interact with many. We study the regime where the coupling is a small perturbation for the former
systems, but a large one for the latter so that global hyperbolicity might be lost. We show that, under some hypotheses on the
heterogeneity of the network, the effect of the coupling on the hub nodes can be well approximated, for a large set of initial
conditions and for a time exponentially long with respect to the connectivity of the hub, by a reduced map of the hub coordinate
only. The map is given by the expectation of the interaction with respect to the invariant measure for the dynamics of the
uncoupled nodes. This implies a bifurcation of the observed behaviour of the hub nodes when varying the coupling strength and
their connectivity and forecast regimes of synchronisation and uncorrelation of the nodes.

29. Shirou Wang, Academy of Mathematics and Systems Science, Chinese Academy of Sciences

Upper semi-continuity of entropy map for nonuniformly hyperbolic systems

For a continuous transformation \( f \) on a compact manifold \( M \), the entropy map of \( f \) is defined by the metric entropy on the set of all \( f \)
invariant measures and it is generally not continuous. However, it is still worth our effort to investigate the upper semi-continuity of it since, for instance, it implies the existence of invariant measures of maximal entropy. We prove that for \( C^1 \) non-uniformly hyperbolic systems with domination, the entropy map is upper semi-continuous. On the other hand we give a counter example showing that this is not necessarily true for \( C^{1+r} \) non-uniformly hyperbolic systems without domination. This goes a little against a common intuition that conclusions are parallel between these two kinds of non-uniformly hyperbolic systems.

30. Ren Yi, Brown University

Self-induced rectangle exchange maps

Rectangle exchange maps (REMs) are higher dimensional generalizations of interval exchange maps (IEMs) which have been well-studied for more than 40 years. We study REMs induced by cut and projection lattices arisen from cubic Pisot numbers. We discover a family of self-induced REMs with fixed combinatorics. We find a Cantor set corresponding to the self-induced REMs in the parameter space and we give a symbolic encoding of their dynamics.

31. Kensuke Yoshida, Tokyo Metropolitan University

Sub-diffusion in a non-chaotic area-preserving map, continuous-time random walks and ergodic problems

We investigate a model of non-chaotic area-preserving map defined on the cylinder that is called the Generalized Triangle Map (GTM). GTM generates sub-diffusion in momentum-direction, and the probability density function of momentum obeys the fractional diffusion equation that describes the sub-diffusion process generated by continuous-time random walk (CTRW) model. In a CTRW, it is shown that the diffusion coefficient of time averaged mean-square displacement (TAMSD) becomes random variable. This situation is called “weak ergodicity breaking” (WEB). On the basis of infinite ergodic theory, it was shown that the diffusion coefficient of TAMSD in dissipative maps with marginal fixed points becomes random variable. This means WEB appears in such maps. We expect that WEB is realized in GTM. However, the phase space is non-compact, so it is not trivial whether the invariant measure exists or not. In this poster, we discuss the description of sub-diffusion in GTM with the CTRW model.

32. Xin Zhang, University of Illinois at Urbana-Champaign

Statistical regularity of Apollonian Gasket

Apollonian gaskets are formed by repeatedly filling the gaps between four mutually tangent circles with further tangent circles. In this paper we study the nearest neighbor spacing and pair correlation of centers of circles from Apollonian gaskets. Unlike many
other point processes investigated in the literature, a key feature in our consideration is that the centers are not uniformly
distributed in any natural ambient space. Nevertheless, after proper normalization, we show that the limiting pair correlation and
nearest spacing exist, and can be described by continuously differentiable functions. These are corollaries of the convergence of
moments that we prove. The input from ergodic theory is an extension of Mohammadi-Oh’s Theorem on the equidistribution of
horospheres in infinite volume hyperbolic spaces, which can be obtained by the mixing of the frame flow under the Bowen-
Margulis-Sullivan measure.

Suggested activities for Wednesday Aug 2nd Afternoon

The conference program for August 2 will end at 3 PM. You have a free afternoon which you can plan as you so wish. Below are some
suggestions**:

a) **Join the planned beach hike**: plan to wear comfortable walking shoes, and plan your own dinner spot. There are a few beach
concessions as well as Jericho Sailing Club- Galley Restaurant along the way (this is an optional hike, see page 20 for details)
b) **Tour UBC**: Nitobe Gardens, the Museum of Anthropology and the Beatty Biodiversity Museum are popular highlights
c) **Take the bus downtown and tour Stanley Park, Canada Place and English Bay**
d) **Go back to your room and rest, nap**

**Please note, that these events are not part of the conference program and are listed here as a suggestion only. For those of you opting
to go for the hike or to tour downtown there will be a fireworks display by English Bay. This is a huge highlight in the Vancouver
Summer and getting back to campus, on transit, may take longer than usual.**
Optional afternoon hike to the Beach

You are invited to a hike through the forest from the Alumni Centre (venue for Remembrance lunch) to the beach. The hike will start at 3PM and take 60-90 minutes. We recommend that you wear good walking shoes and bring water. At the beach, there is food available for purchase at beach concessions or at Galley restaurant (Jericho Sailing Club), marked on map below. Please note that the concessions and Galley restaurant cannot accommodate us all at the same time. Also, there will be fireworks at 10PM over English Bay; best viewing spots are Vanier Park (near Granville Island) and Sunset Beach (other side of Burrard Bridge).

Hike route below: (There are other possible routes; however, note that high tide will be at 4:30 PM.)

1. Walk east on University Blvd and enter forest at Sword Fern Trail (24)
2. Sword Fern Trail to Spanish Trail (23) and go east
3. Spanish Trail to Pioneer Trail (17) and go north [be careful crossing Chancellor Blvd]
4. Pioneer trail to East Canyon Trail (7) and go west and then north; East Canyon trail to Admiralty Trail (1)
5. Go east on Admiralty Trail to NW Marine Drive
6. Cross NW Marine Drive to beach and continue east to find food at concession or sailing club.

Start of hike: Sword Fern

- Spanish Banks West Concession
- Spanish Banks East Concession
- Locarno Concession
- Jericho Sailing Club-Galley Restaurant