As Geometry is lost
What connections are lost?
What reasoning is lost?
What students are lost?
Does it matter?

Walter Whiteley
York University
Graduate Programs in Math, in Education, in Computer Science, in Interdisciplinary Studies

Outline

- Where I come from
- Learning Geometry - from early childhood

  and from ‘Learning to See’
- Needing Geometry in other areas
- Sample Geometry investigations for

Where I come from

- learned geometry after my studies
- 20 years as a CEGEP teacher (pre-university)
- 35 years as a researcher in Applied Geometry
- 15 years teaching geometry to future teachers, in-service teachers
- Active researcher on visual, kinesthetic reasoning
- I now see geometry everywhere

Where I come from

- Involved in last curriculum revision in Ontario
- Chaired university input group
- Observed the pressures to remove geometry
- Dominance of the push to calculus
- but calculus without the geometry
- See geometry everywhere except in grades 10-14!

Some of my wildest dreams
are about geometry!

Early childhood mathematics and Mathematical Cognition

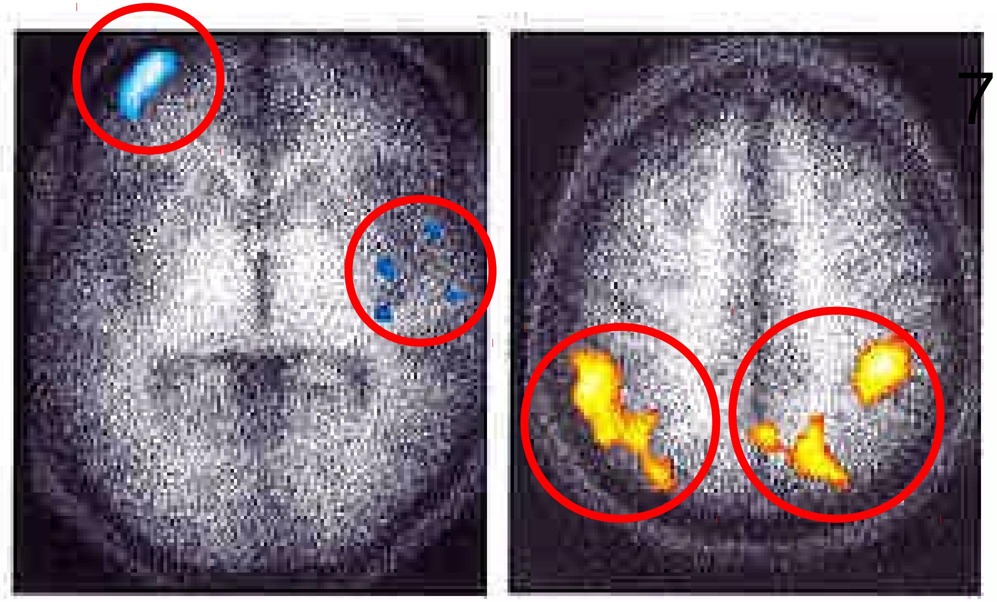
- Number
- Geometry
- Metaphors
- Spatial perception: you create what you see

Number Sense

- Number sense(s)
- Subitizing (small numbers)
- Comparing larger numbers
- Number line in our neural networks
- Calculating with language

Adults with numbers

$7 \times 5 \; = ?$



$7 \times 5 \; > 26 \; ?$

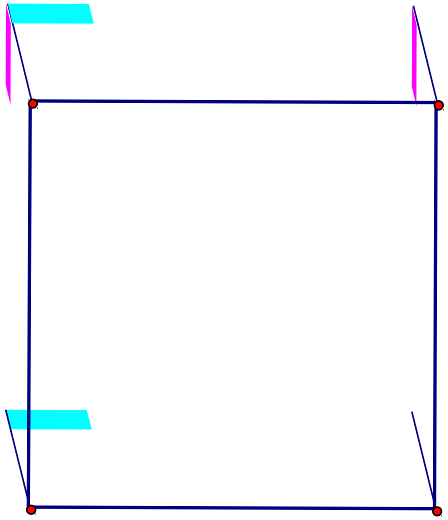
Language (tables)

Visual (analog number line)

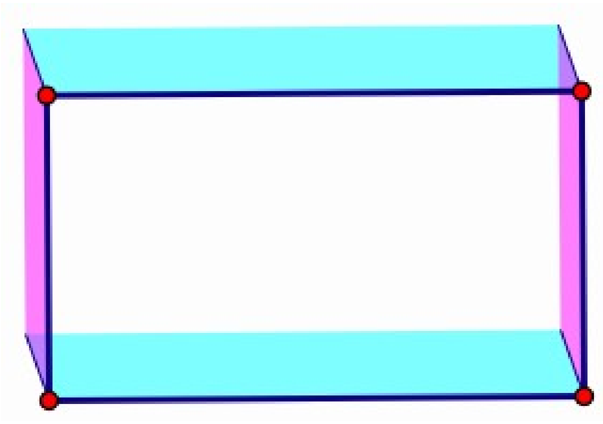
Early childhood Geometry

- Geometric Sense(s)
- Elizabeth Speilke: Beyond core knowledge
- Navigation / location in larger spaces
- Finding a hidden object in an enclosed room
- Square room vs rectangular room vs rhombic room

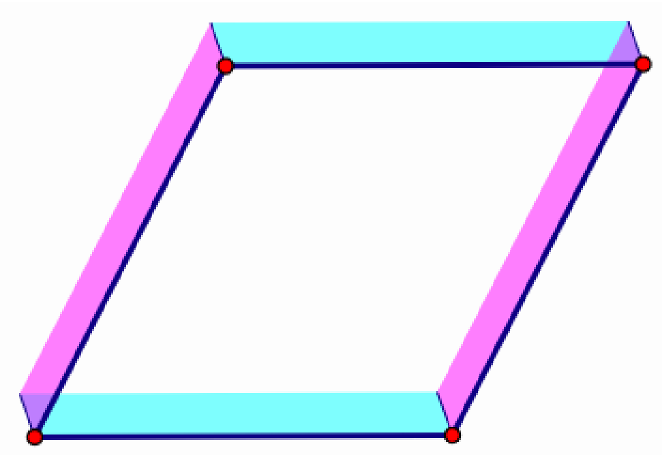
Search in corners of a room:



Square: all 4 corners



Rectangle: 2 corners

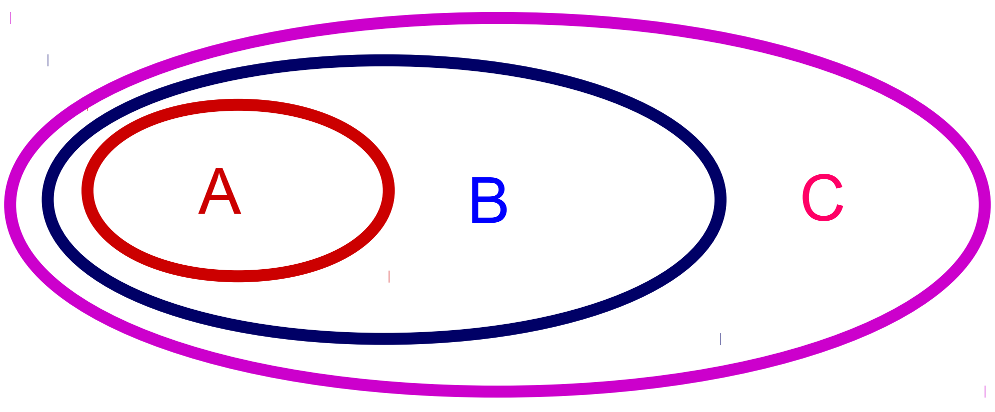


Rhombus: ?

- Sense of length (3-D)
- Start without sense of angle (<4 years)
- Not effective with 2-D cues

Metaphors and Transfer

- Consider:
- All A are B
- All B are C
- Therefore All A are C
  - Is this language based?

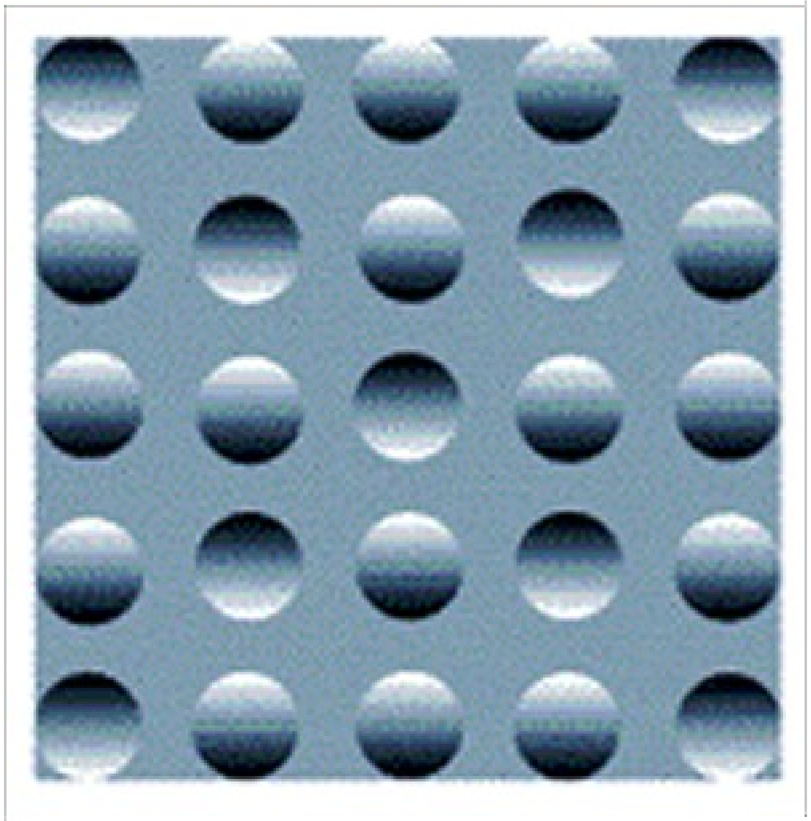


No “containers”

Vinod Goel,

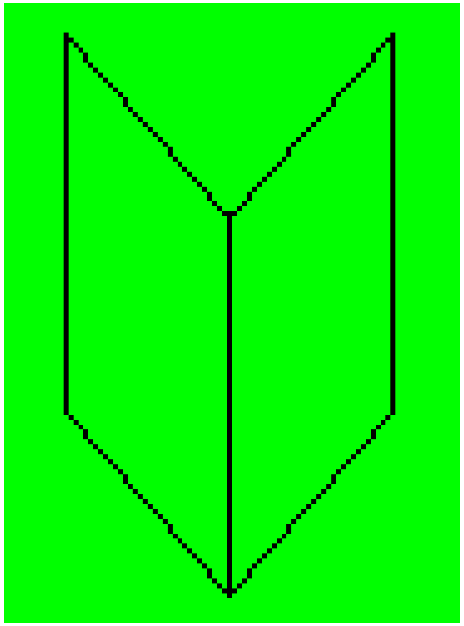
Lakoff and Nunez

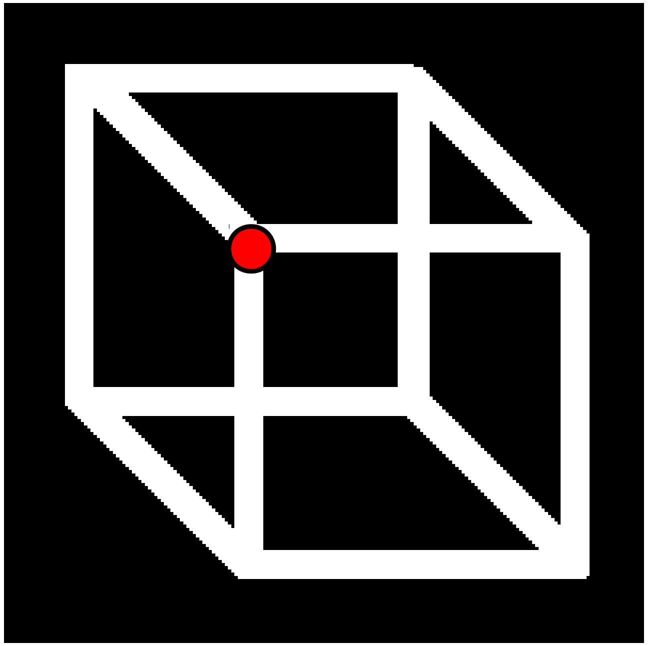
- We create what we see



We assume that light comes from the top

- 2-D pictures of 3-D are ambiguous.
- we may “flip” from one view to another.





Necker Cube

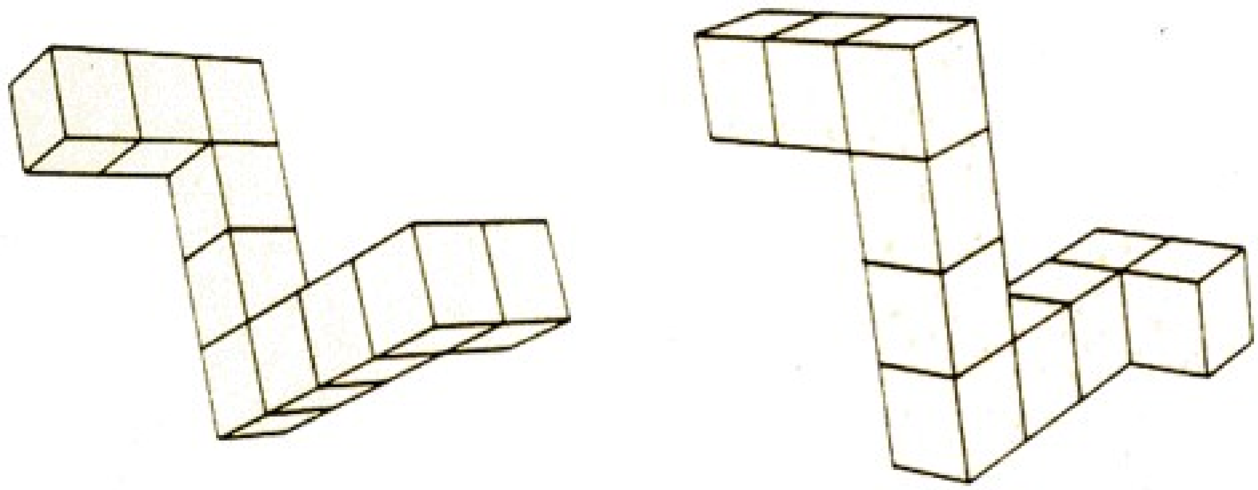
Do you see what I see? No!

We process, select, and construct what we see.

We can change what we see

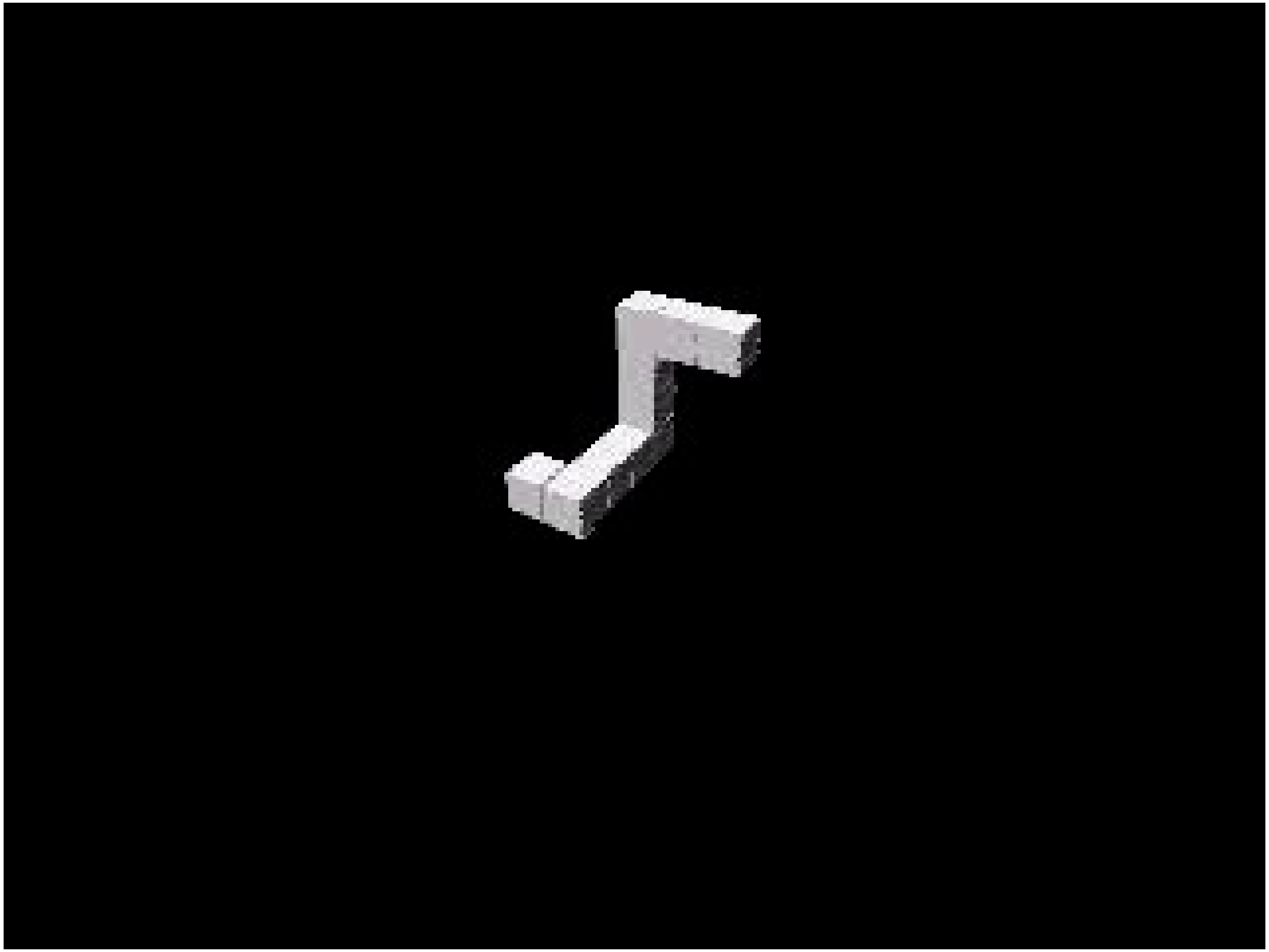
- In mathematics we create:
  - from experience: eyes and hands
  - from practice and apprenticeship
  - from insight and understanding.

Mental Rotation



Friedhoff

Mental Rotation



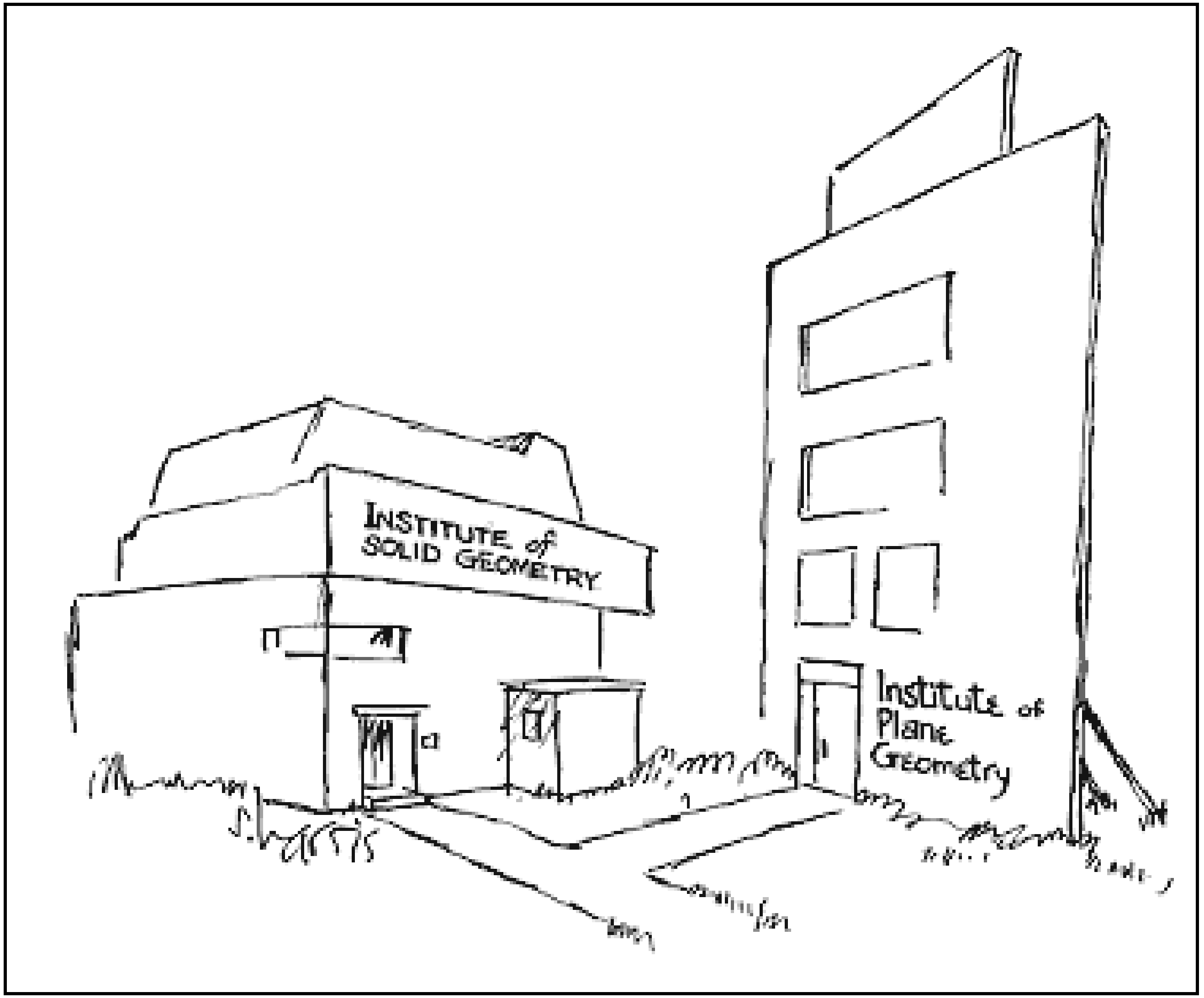
More Early Geometry

- Mirror Neurons (actually rotation)
- Connectivity - what can I reach?
- Work of Doug Clements (pre-K to 3)
- Need for sample space - with variations
- Value of non-examples
- Orientation of shapes: Australia and picture books
- Anticipate a rich set of abilities cognitively linked to geometry

“Current evidence suggests that mathematical abilities are associated with the evolution of eye-hand coordination and the manipulation of objects. … Far from being a language, mathematics represents a thoroughly independent and powerful mode of brain function.”

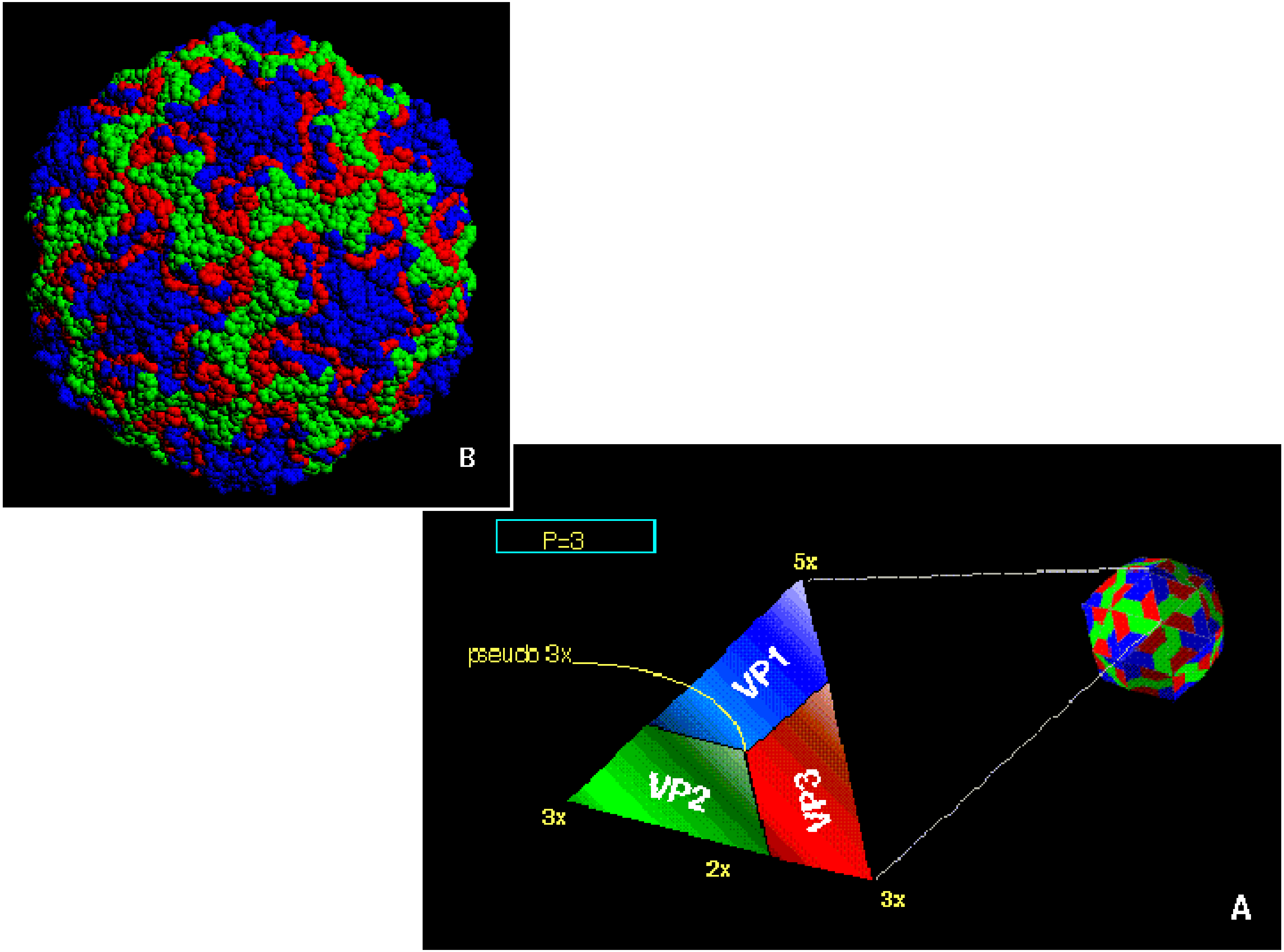
Hugh R. Wilson (York) cognitive scientist

3-D before 2-D



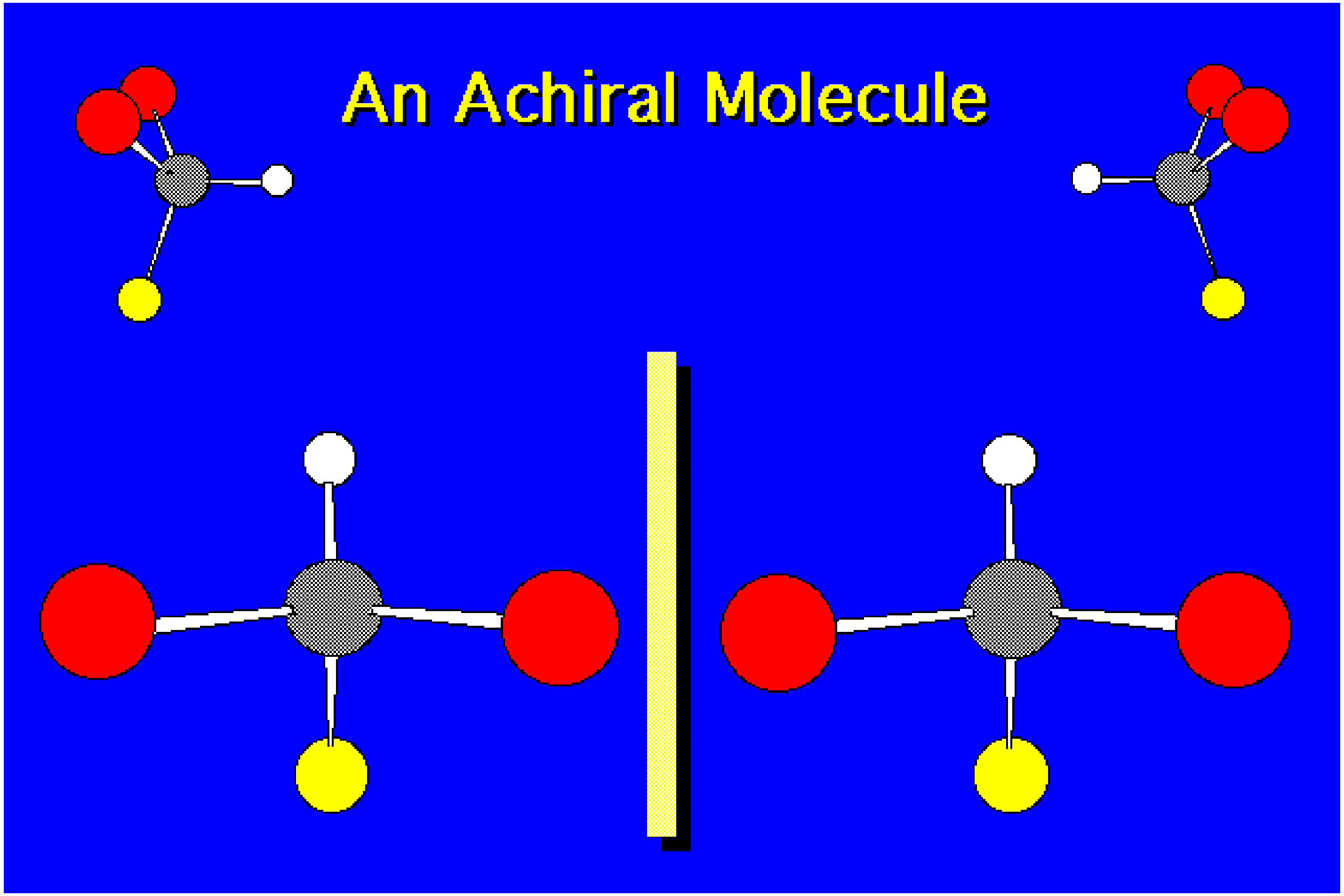
Geometric reasoning across fields

Biochemistry
Physics
Computational geometry,
Robotics, computer games, computer vision,
Visual / Spatial reasoning
data visualization, YouTube …
embodied, human, ‘sensible’ skill

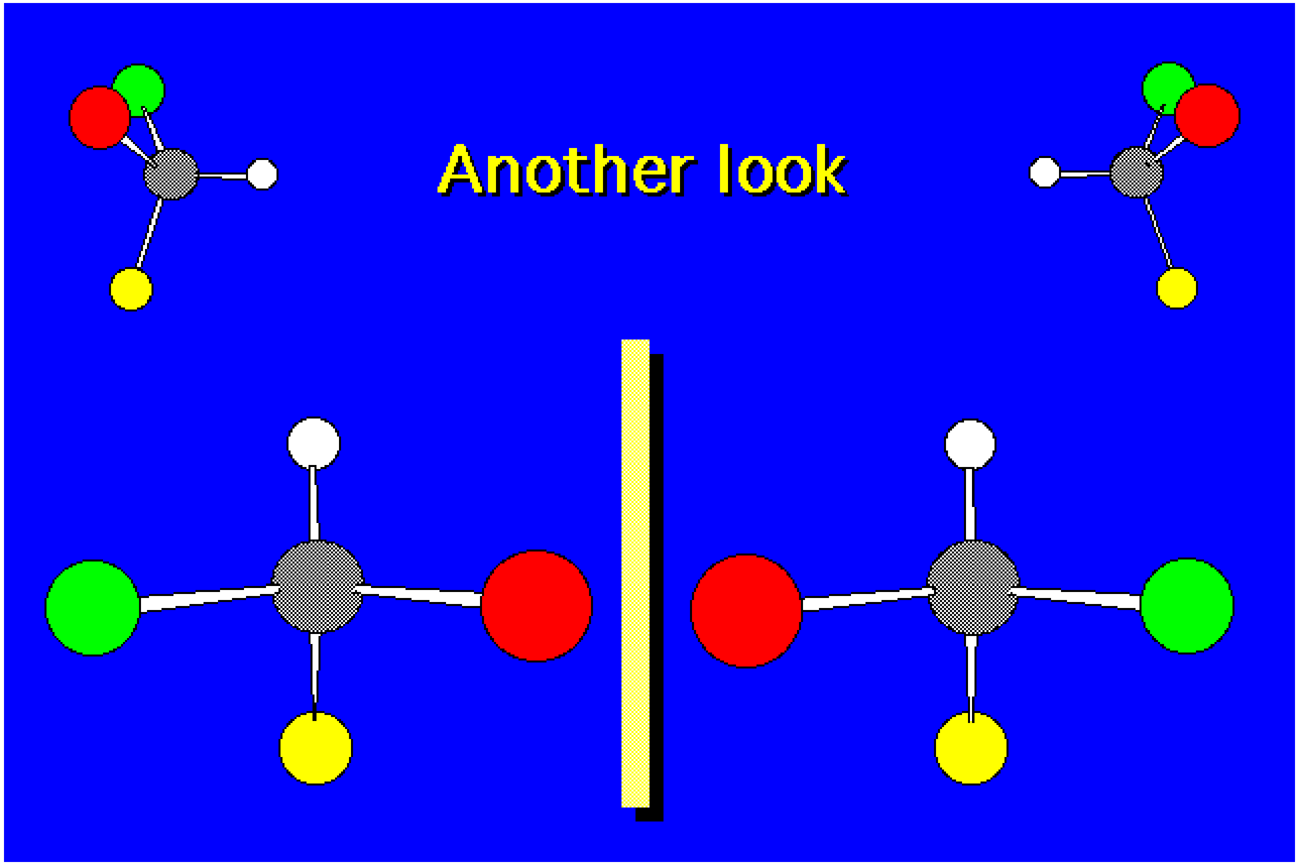


B
P=3
5x
pseudo 3x
VP1
VP2
VP3
3x
2x
3x
A

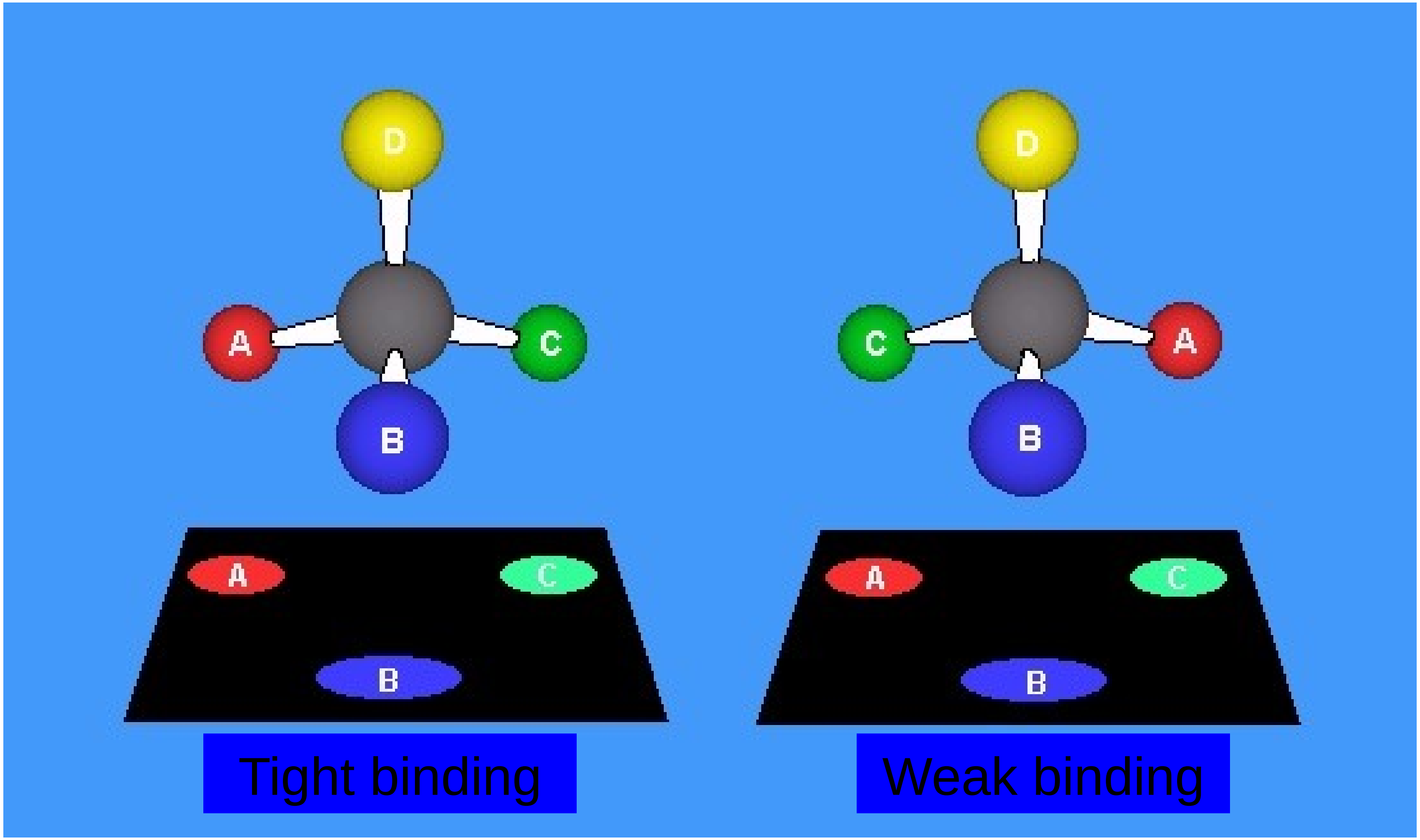
- Symmetry is central to Stereo Chemistry
- Achiral: Mirror image is the same as original
- Can you tell, looking at one copy of the molecule?



- Chiral (handed): Right and Left hand versions are different
- Spearmint vs Caraway - same molecule



- Chiral binding (drugs)
- Thalidamide …

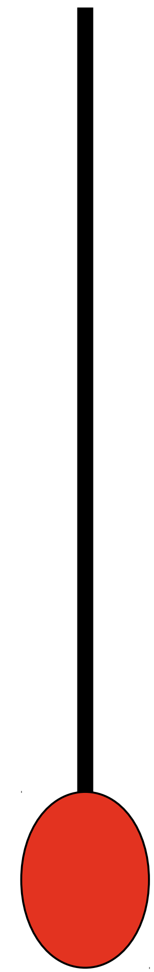


Pierre Cure’s Principle (1894)

The symmetry of the input appears in the symmetry of the output.

The hanging weight on a string

- Why straight?
- Symmetry of forces
- Straight (geodesic) is curve with key symmetries
- Sufficient reason:
- No reason to bend, therefore straight.



Noether’s Principles

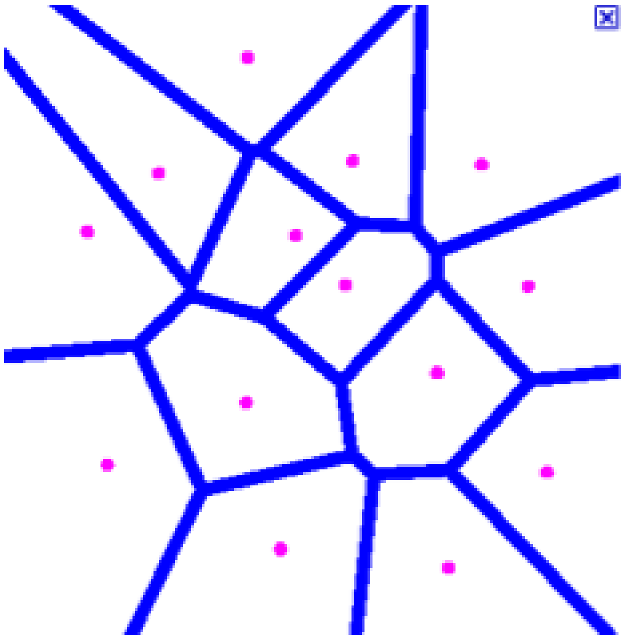
Every symmetry in the laws of physics generates a conserved quantity
Every conserved quantity in physics corresponds to a symmetry in the laws of physics.

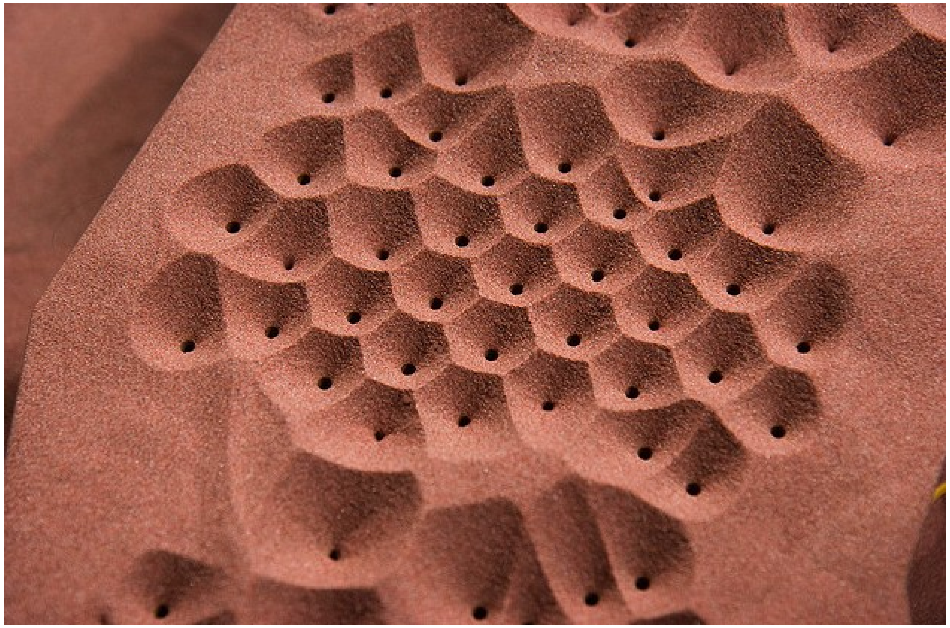
Noether's Principles (cont)

The laws of physics are the same today as yesterday and tomorrow
Conservation of energy
The laws are the same here in Toronto as across the room or in Vancouver
Conservation of momentum
The laws are the same facing west as facing north
Conservation of Angular Momentum

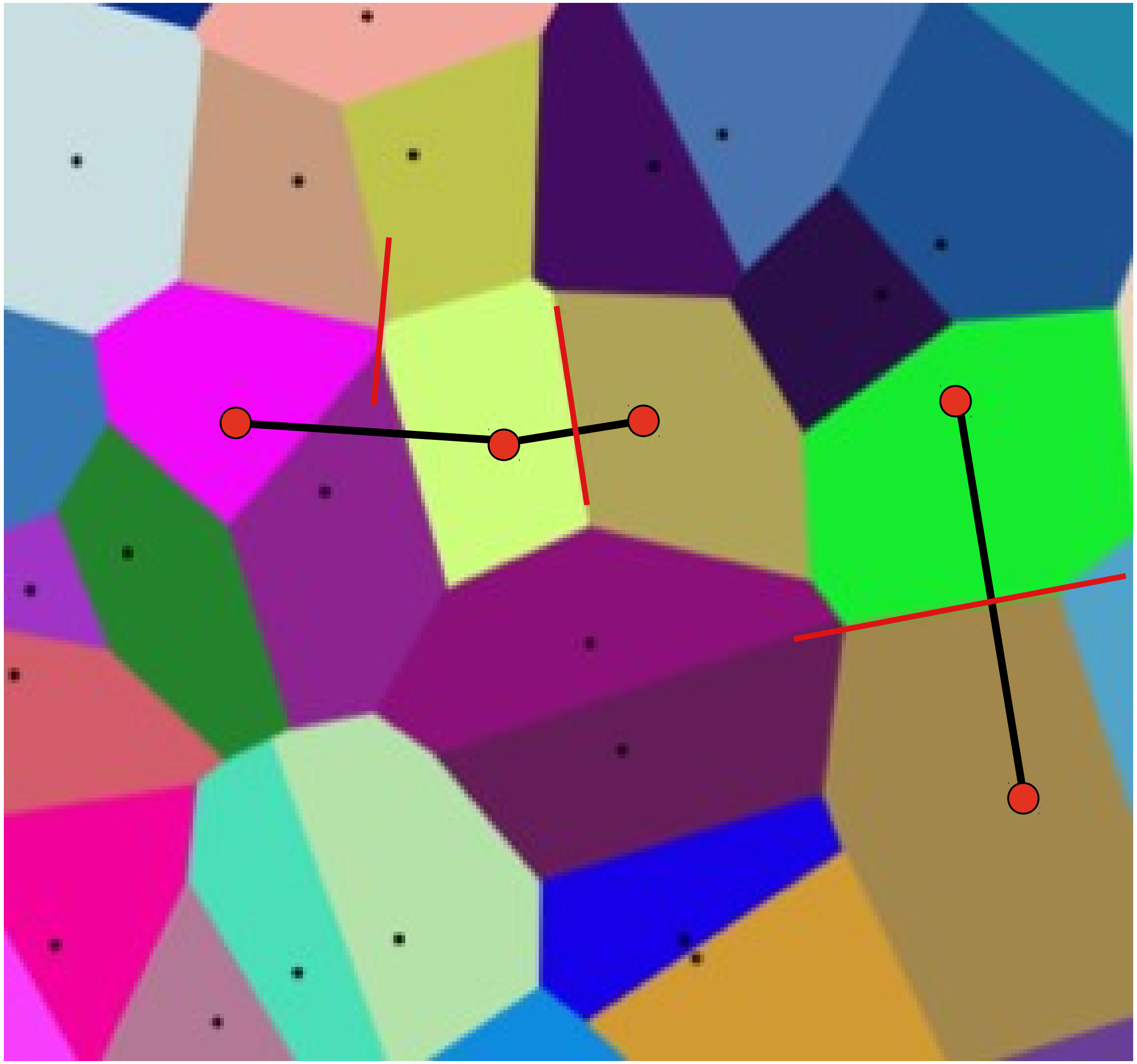
Computational Geometry

Voronoi Diagrams
A core area: algorithm, data structure, …
Regions closer to seed point than to any other.



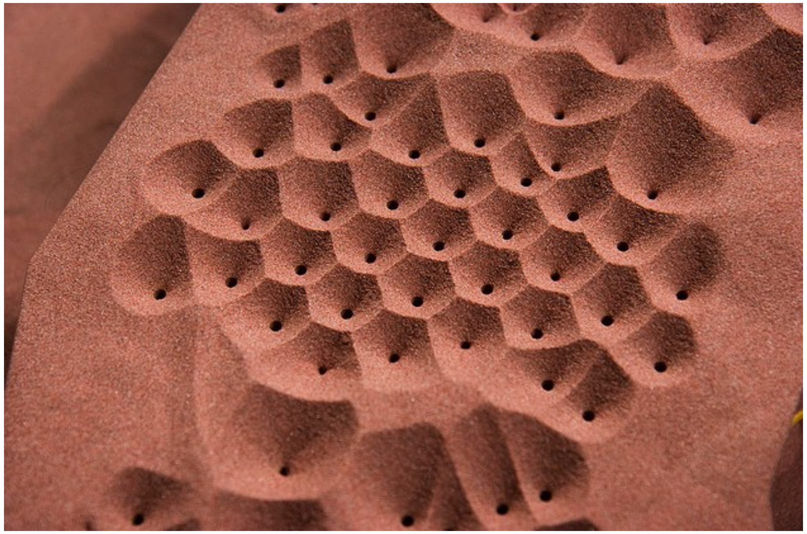


Voronoi Diagram



Voronoi Diagrams (cont)

The right bisectors of the edges joining the holes
Connects to right bisectors of the edges of a triangle.



Right Bisectors

Voronoi Movie

- Also curves equidistant between a point and a line …

Geometry is Everywhere

Key to modern practices – is symmetry and transformations
1794 – modern version of symmetry
Groups of transformations, reasoning with transformations
Altered scrutiny – see the world differently
Bring that reasoning into mathematics and science classes.

Bridging the Geometry Gap:
Early Childhood to later reasoning

What we have by age 12 (age 6?) is ‘schooled’
Different for those without any schooling
Use it or lose it
Do not effectively use it in school
Cognitive Pieces are integrated / blended in individual ways
Ability to use multiple approaches and switch (rapidly) is mark of exceptional students.
People who rely essentially on visual / kinesthetic geometric reasoning.

Solving Problems with Symmetry

using symmetry and transformations in problem solving.
Here is an example.

Square Dissection

- Yalgom: Geometric Transformations (MAA Press), four volumes, high school
- in Russia!

Affine transformation: the sun (or parallel rays).

- Sheering
- Stretching one axis

- Preserves parallels
- Preserves ratio of areas

Can you transform any triangle into an equilateral triangle by affine transformations?

Solving Problems with balance

Center of mass and centroid
Consider the reasoning with weights

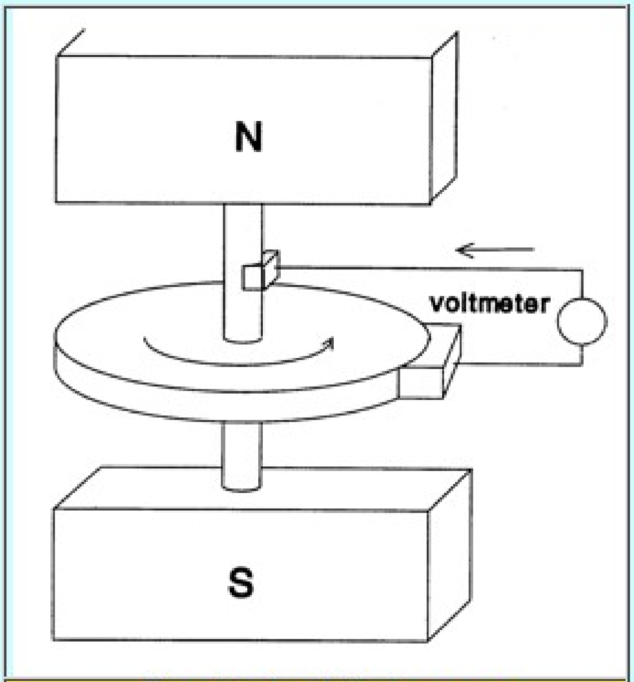
Median Balance

- Statics is affine

Who is lost?   Michael Faraday

Evidence of dyslexia, dyscalculia: did not, could not, use formulas
Visual reasoning – notebooks the day he built the first electric motor

- Was this mathematics?
- Will it matter if we exclude future Faradays?



Faraday's disk dynamo

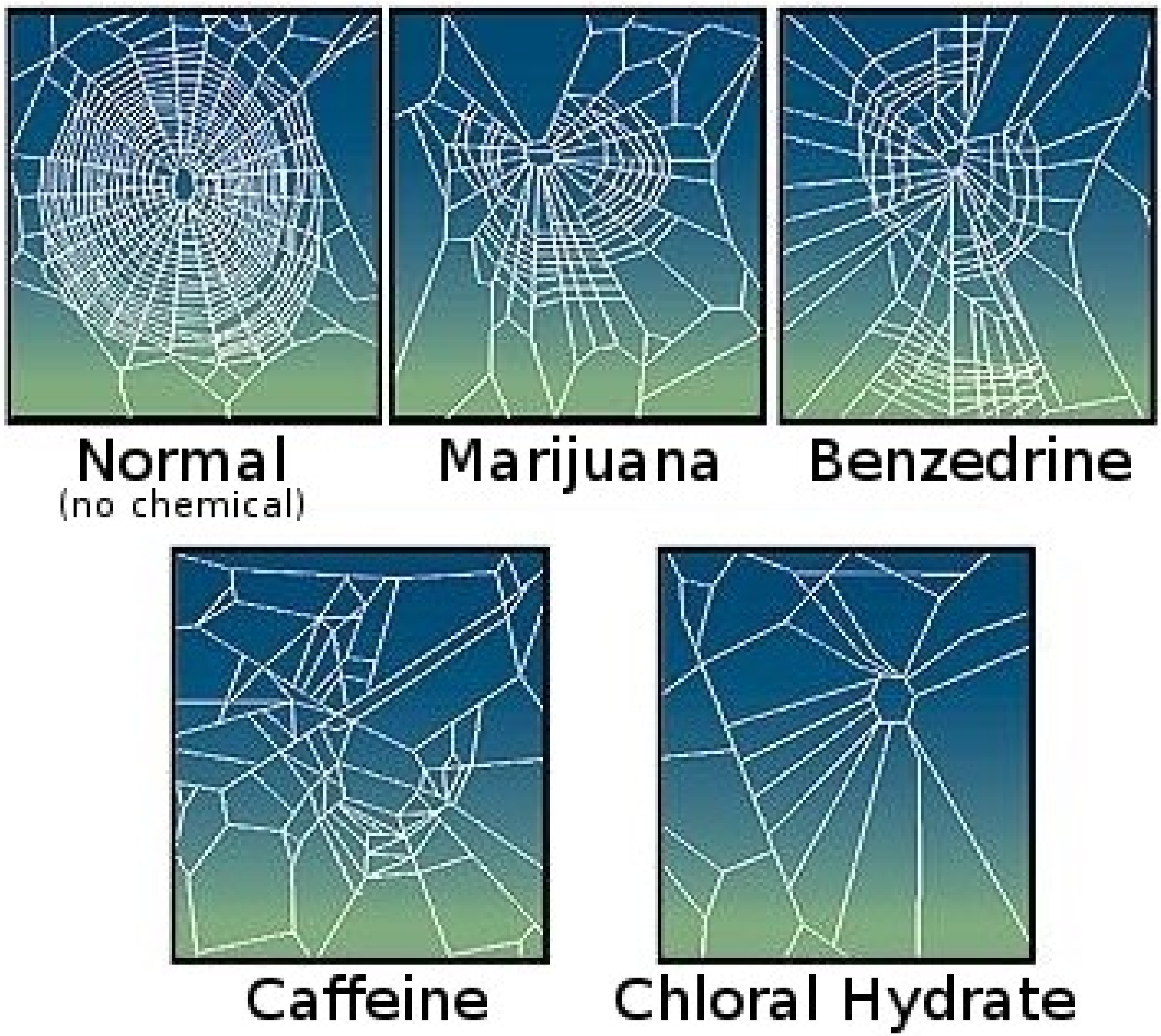
Michael Faraday (cont)

As I proceeded with the study of Faraday, I perceived that his method of conceiving phenomena was also a mathematical one, though not exhibited in the conventional form of symbols. James Clerk Maxwell

Does this Geometry Gap matter?

- Learners pushed out of math,
- missing the opportunity to succeed
- ‘Successful students’ - miss additional skills and flexibility
- Shock points (Calculus III – multiple integration)
- Vital skills / connections in other subjects
- Compare to countries outside North America.
- I see geometry everywhere
- My students deserve a chance to learn this.

Geometry on drugs



Thanks

Questions

whiteley@yorku.ca

More Spiders