

2009 Graduate Industrial Mathematical Modelling Camp – Mentor Problem Statements

Pacific Institute for the Mathematical Sciences (PIMS) www.pims.math.ca/industrial/workshops/
2009 Graduate Industrial Modelling Camp and Industrial Problem Solving Workshop
University of Calgary, May 19-29, 2009

Brian Alspach, University of Newcastle, Australia



Brian Alspach has published approximately 100 research papers in graph theory and combinatorics since 1967. In addition, he has written about 200 magazine articles on the mathematics of poker. He has served or is serving on the editorial boards of *Ars Combinatoria*, *Australian Journal of Combinatorics*, *Canadian Journal of Mathematics*, *Canadian Mathematical Bulletin*, *Discrete Mathematics*, and the *Journal of Graph Theory*. The three issues of Volume 299 (2005) of *Discrete Mathematics* were entitled "The Graph Theory of Brian Alspach" and devoted to papers dealing with his work.

He was the project leader of a MITACS (Canada) project with an annual budget of \$150,000. The project dealt with network security. He also held NSERC grants for the period 1967 - 2008. He has many years of experience doing industrial consulting specializing in scheduling problems and the gaming industry.

OPTIMAL SEARCHES AND PAIRINGS

Problem 1:

Suppose we are given m job applicants and N companies, where the companies have a_1, a_2, \dots, a_N job openings, respectively. Each applicant has an interview with some of the companies and each company interviews some subset of the applicants. The companies and applicants provide rankings of their various candidates/potential employers. Look for optimal pairings of applicants to positions under varying conditions on m, N and the ranking scheme. It is probably advisable to start with a simple model and add sophistication as warranted.

Problem 2:

A spelunker is lost in a complicated cave system. Find an optimal search strategy under various constraints such as a limited number of searchers, limited time to find the spelunker, the spelunker is allowed to move at different speeds, and so on.

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Mik Bickis, University of Saskatchewan



Mikelis Bickis received a BSc degree from the McGill University, and a MSc from the University of Alberta. He worked for fifteen years as a statistician with the federal government, in the areas of agricultural research and health protection. He received his PhD from Carleton University and for the last 23 years has been on the faculty of the Department of Mathematics and Statistics at the University of Saskatchewan.

Dr. Bickis teaches courses in biostatistics, experimental design, and regression analysis, as well as in probability and statistical theory. He is currently supervising two graduate students. Under his supervision, seven masters and two doctoral students have already earned their degrees.

Often consulted about problems in statistical analysis or design (mostly in life science applications), Dr. Bickis is currently engaged in collaborative research in bioinformatics, specifically in the distribution of oligopeptides in proteomes, having implications in immunology and vaccine design. He is also doing theoretical research, focusing on problems of multiplicity in statistical inference.

In both his teaching and research, Dr. Bickis capitalizes on geometrical connections to statistical concepts. He has worked on interactive computer graphics to help with the visualization of statistical information. <http://math.usask.ca/~bickis>

VISUALIZATION OF SETS OF CONDITIONAL INDEPENDENCE STRUCTURES

In the statistical study of relationships among several variables, it is useful to model the conditional independence structure of the system. Such a structure can be deduced from causal relations between the variables, and conversely, causality can sometimes be inferred from the conditional independence structure. Typically, such relations are represented by directed or undirected graphs in which the nodes represent variables and the edges represent a causal link or association. Conditional independence relations in such graphs are described in terms of separation properties of the graph. A Bayes net, much used in artificial intelligence implementations, is an example of such a graph.

The structure and analysis of such graphs have been well studied in the mathematical, statistical, and computer science communities in the last couple of decades. Much less studied, however, is how the set of such structures sit together in the collection of all models. For the inferential problem, it is important to understand how "close" alternative structures are in the space of probability distributions, for without some such metric it is not possible to gauge the evidence that an empirical data set provides against or in favour of competing models.

The goal of this project is not so much to examine such metrics mathematically, but to provide visualization tools for stimulating intuition about these relationships. We are interested in producing interactive computer graphics that display some or all of the geometric relationships between different conditional independence structures inherent in the space of models. Since all but the simplest structures are high-dimensional, the challenge is to create graphics that capture

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essential properties of these multidimensional structures in an apparently three-dimensional image projected onto a flat computer screen.

The simplest non-trivial case involves three variables with an assumed Gaussian distribution. In this case, the conditional independence structure is characterized by the correlation matrix. A correlation matrix is defined by the three entries in the upper triangle and hence can be represented as a point in three-dimensional space, with the constraint that the resulting matrix be positive semi-definite. There are five classes of conditional independence structures in this case:

1. Total dependence
2. Total independence
3. Marginal independence between two variables dependent with the third
4. Conditional independence between two variables given the third
5. Marginal independence between each of two variables and the third in addition to conditional independence given the other.

In each of the last three classes there are three choices of the pair of variables, leading to a total of eleven possible structures among the three variables. Each of these structures corresponds to one or more equations involving the correlations, and thus can be represented by a set in three dimensions.

The first phase of the project would require identifying the set of points in three-dimensional space corresponding to correlation matrices, as well as the subsets corresponding to the different conditional independence structures. A computer implementation of these sets would be developed, with a facility for interactively highlighting, selecting, and/or hiding various subsets. Motion and lighting effects could be utilized to enhance the visualization.

The next level would consider three binary variables. The set of possible distributions is a seven-dimensional simplex. However by fixing the expectation of each variable to $1/2$, one can focus on a four-dimensional subset which still includes all the possible conditional independence structures. There are conditional independence structures possible in this situation which cannot occur in the Gaussian case. One would need to determine all the possible conditional independence structures and the subsets of the model space to which they correspond. These would then need to be implemented as interactive computer graphics that allow the visualization of a four-dimensional object and its subsets. Going into the other dimension requires some creativity to achieve a persuasive visualization.

Going to more than three variables multiplies the dimensions. A four-variable Gaussian correlation matrix lies in six dimensions. Four binary variables with equiprobable margins require eleven dimensions. Any ingenious ideas that would allow one to visualize such structures would be very innovative.

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Neville Fowkes, University of Western Australia, Australia



Dr. Fowkes completed his Ph.D. at the University of Queensland in Australia specializing in applied mathematics (asymptotics) and then worked at the Division of Engineering and Applied Physics at Harvard University. It was there that he became interested in modelling. And later took up a position at the University of Western Australia.

Dr. Fowkes is a mathematical modeller who works mainly on continuum problems arising out of industrial, scientific and biological areas. He has participated in more than 30 Maths in Industry Study Groups (MISGs) held in many different countries. Companies he has worked with include Uncle Toby's, BHP, Hardie, CRA, ICI, DuPont, British Steel, BrewTech, Mouldflow, Age Developments, Unilever, LNEC, Petronas, SAB. Dr. Fowkes has helped initiate (and facilitate) MISG's in Indonesia, S. Africa, and has assisted in the development of industrial mathematics graduate and undergraduate programs in Portugal, China, Indonesia, Laos and Thailand. He has authored a text on mathematical modelling based largely on problems arising out of my industrial mathematics experiences and has been used in industrial mathematics courses in Australia, the UK and Europe.

THE MOORING OF LARGE SHIPS

A ship moored at a wharf or anchored to the ocean floor may be acted on by a variety of dynamic environmental forces such as waves, winds and currents. These forces cause the ship to undergo motions which, if sufficiently large, may adversely affect port operations (loading and unloading) or may result in failure of the mooring lines or damage to wharf and ship, or loss of life. In the case of the giant 'mother ship platforms' used for oil storage out at sea the flexible hose connecting the ship to the well-head may break causing an environmental disaster. We would like to understand the dynamics of such moored ships so that better safety measures can be taken.

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Paul McNicholas, University of Guelph



Paul McNicholas is an Assistant Professor at the University of Guelph's Department of Mathematics and Statistics. Paul was educated at Trinity College Dublin, Ireland, where he read mathematics (B.A., M.A.), high-performance computing (M.Sc.) and statistics (Ph.D.). His research group at the University of Guelph currently consists of six graduate students and a number of research assistants.

The main focus of Paul's research is model-based clustering, although work is also being carried out in other areas of application of mixture models and in data mining. Following the development of new techniques for model-based clustering, these techniques are implemented efficiently in parallel. To date, two new model-based clustering techniques have been developed: one specifically designed for longitudinal data and one more generally applicable technique that can be viewed as an extension of the mixtures of factor analyzers model. Areas of application include bioinformatics and food authenticity. Other recent work has focused on the estimation of cure rates in survival analysis and on developing new measures of the interestingness of an association rule.

For further information, you can visit Paul's website at www.uoguelph.ca/~pmcnicho.

AUTOMATIC CLASSIFICATION & DATA REDUCTION FOR FOOD AUTHENTICITY PROBLEMS:

Foods are sometimes passed off as being something other than what they actually are. In order to determine if a given food is actually authentic, it is necessary to take a number of physical and chemical measurements. Such measurements have a cost attached to them, both in time and money, and it is generally desirable to take as few measurements as possible. The creation of an 'automatic' technique that will both devise an efficient classifier and produce a reduced set of necessary measurements would be of great use to the scientific community. The challenge is to create such a technique and demonstrate its effectiveness on three real data sets: one on wines, one on olive oils and one on maple syrup.

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Sévérien Nkurunziza, University of Windsor



Dr. Nkurunziza has a Ph. D in Statistics since 2005. His research interests are in the areas of mathematical statistics, mathematical modeling, applied probability and inference in stochastic processes with their applications to biological/ecological systems, and quantitative finance. In particular, Dr. Nkurunziza is interested in statistical inference and modeling of ecological systems such as predator-prey species. He also works on stochastic modeling of the term structure of interest rates and on inference for the drift parameters of diffusion processes. Further, his current research includes inference for stochastic processes with regime-switching, asymptotic properties of estimators and large-sample inference, shrinkage and pretest strategies, and generalized inference

MODELING AND INFERENCE PROBLEMS IN SOME MULTIVARIATE ECOLOGICAL SYSTEMS OR QUANTITATIVE FINANCE

We consider modeling and inference problems in some multivariate ecological systems or quantitative finance. In particular, we will study the case of p -dimensional stochastic process (with $p > 2$) that models the evolution of multiple interacting species or multi-factor interest rate structure. More precisely, we will focus on the cases of p -dimensional ergodic diffusion process where the drift parameter is suspected to lie in a certain hyper-plan. Under such uncertainties, classical maximum likelihood estimator (MLE) loses in efficiency. We will present robust inference technics for improving the performance of classical maximum likelihood estimator. Further, we will discuss some asymptotic properties and will give illustrative numerical results based on simulations studies. Finally, we will present an algorithm for performing Monte Carlo simulations and the participants will be invited to execute the algorithm by using Matlab software.

In summary, the presentation will cover the following topics.

1. Modeling problem in some ecological systems and in quantitative finance;
2. The Radon-Nicodym likelihood function of ergodic diffusion process;
3. The unrestricted and restricted maximum likelihood estimator of drift parameter (MLEs);
4. Asymptotic normality of the MLEs ;
5. Robust inference strategies in diffusion processes;
6. Asymptotic distributional risk and relative efficiency;
7. Monte Carlo simulations.

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Michael Slawinski, Memorial University



Dr. Slawinski's research focus is quantitative formulation of properties describing seismic phenomena. He is a professor of seismology at Memorial University and an adjunct professor of mathematics at the University of Calgary. Presently, he is a visiting professor at Princeton University. Recent editions of his two books, which are available online and a selection of papers that are representative of both his research and the proposed problem are listed below.

- Slawinski, M.A. (2007) *Waves and rays in elastic continua*. Samizdat Press:

- <http://samizdat.mines.edu/wavesandrays/> 470 pages

- Bóna, A., Slawinski, M.A. (2007) *Ray theory: Characteristics and asymptotics*.

- Samizdat Press: <http://samizdat.mines.edu/characteristics/> 199 pages

- Kochetov, M., Slawinski, M.A. (2008) On obtaining effective transversely isotropic elasticity tensors.

- Journal of Elasticity* 94(1), 1–13

- Bóna, A., Bucataru, I., Slawinski, M.A. (2007) Coordinate-free characterization of elasticity tensor.

- Journal of Elasticity* 87(2-3), 109–132

- Bóna, A., Bucataru, I., Slawinski, M.A. (2004) Material symmetries of elasticity tensor. *The Quarterly Journal of Mechanics and Applied Mathematics*

- 57(4), 583–598

MATERIAL PROPERTIES FROM WAVE-PROPAGATION MEASUREMENTS:

It is common to assume that the actual material composing the Earth can be described by an idealized medium whose relation between stress and strain is linear. Such a medium, which belongs to the realm of continuum mechanics, is called a Hookean solid in honour of Robert Hooke, who introduced this concept in the seventeenth century. In spite of its simplicity, which is in agreement from Occam's law of ontological parsimony, this first-order approximation leads to a plethora of mathematically challenging and physically relevant problems. A Hookean solid is described by twenty-one parameters. Evaluating these parameters from measurements is the problem that we are facing. One could follow a trial-and-error approach: postulate a set of parameters, calculate the expected observables, compare the calculated values to measurements, and repeat the process until a satisfactory agreement between the model and observations is achieved. We will follow an approach of inferring these parameters from measurements that is both more challenging mathematically and more pertinent physically: an inverse problem. In studying this inverse problem, we will consider aspects of the theory of partial differential equations, tensor analysis, group theory, and matrix theory. We will encounter physical concepts of seismic disturbances, and abstract notions of distance in the space of tensors, since to achieve an accurate representation of the physical world, we have to invoke advanced mathematical concepts.

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Yuriy Zinchenko, University of Calgary



Yuriy Zinchenko received his PhD in Operations Research from Cornell University in 2005. Since then he held a post-doctoral position at the Advanced Optimization Lab, McMaster University. In 2006 Yuriy also joined the Department of Radiation Oncology, Princess Margaret Hospital, as a post-doctoral researcher, and received 2007 MITACS Best Technology Transfer award for his work in optimal radiotherapy design. In August 2008 Dr. Zinchenko joined the Department of Mathematics and Statistics, University of Calgary. His research interests include: mathematical programming with applications to computational geometry, operations research, optimization algorithms and software, scientific parallel computing and high-performance linear algebra, applications to medicine and healthcare, and optimal radiation therapy design for cancer treatment.

MOMENT PROBLEM IN THE CONTEXT OF RADIOTHERAPY PLANNING

More than half of cancer patients receive radiation therapy as part of their treatment. In external-beam radiation therapy, a patient is typically positioned under a robotic arm of a linear accelerator that delivers radiation. High energy radiation has the capacity to destroy cancerous cells; however, the radiation also damages healthy tissues. Therefore, a strategic radiation delivery is required to conform to the shape of the tumor while delivering the minimum amount of radiation to the surrounding healthy organs.

Planning for radiation therapy involves solving an inverse problem. The inverse problem attempts to find the best match between physically-realizable treatment plans and the ideal treatment plan prescribed by a physician. Optimization models and methods are commonly used to accomplish this task.

The simplest model for radiotherapy planning may be formulated as a system of linear inequalities, where radiation dose delivered to the tumor has to exceed a certain *lethal* threshold, while dose to healthy tissues must not exceed some *critical* threshold value. In this case our goal would be to identify a feasible solution or to prove that no such solution exists. One can formulate a variant of least-squares problem (with additional non-negativity constraints) to answer these questions.

Most numerically attractive optimization problems are the so-called *convex* problems, such as minimizing a linear functional over a convex subset of finite-dimensional real vector space. On the other end of the spectra, optimization problems that lack convexity are notoriously difficult (and often even impossible) to solve numerically.

The so-called *dose-volume distributions* form a set of particularly important treatment planning criteria that captures tissue's biological response to radiation. Conventional modeling techniques applied to these dose-volume distributions result in non-convex optimization problems. As a result, practically such optimization problems may be handled only with super-computing resources.

Our goal is to investigate alternative convex reformulations and relaxations of dose-volume requirements.