

Problems, May 2009

Problem 1. Suppose that

$$P(x) = \frac{a}{3!}x(x-1)(x-2) + \frac{b}{2!}x(x-1) + \frac{c}{1!}x + d.$$

Show that $P(n)$ is an integer for every integer n if and only if all of a , b , c , and d are integers.

Problem 2. Let n be odd. Show that if an n -gon inscribed in a circle has all its angles equal, then all its sides are equal.

Problem 3. Call a triangle *accidental* if its area (in units²) is numerically equal to its perimeter (in units). (a) What is the smallest possible value of the perimeter of an accidental right-angled triangle? (b) What is the smallest possible value of the perimeter of an accidental triangle?

Problem 4. Let the sequence a_0, a_1, a_2 , and so on be defined by $a_0 = 1$, and

$$a_0a_n + a_1a_{n-1} + a_2a_{n-2} + \cdots + a_na_0 = 1$$

for all $n > 0$. Find an explicit expression for a_n .