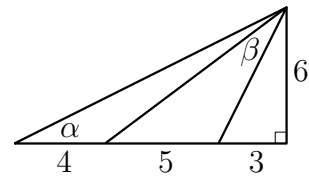


## Problems, March 2009

**Problem 1.** In the right-angled triangle below, dimensions are as shown. Prove that  $\alpha = \beta$ .



**Problem 2.** (Please note that the originally posted version had two typos.) Let  $f(x) = ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are real numbers. Define  $f'(x)$  by  $f'(x) = 2ax + b$ . Show that if  $|f(x)| \leq 1$  on the interval  $[-1, 1]$ , then  $|f'(x)| \leq 4$  on the same interval. No calculus please!

**Problem 3.** Show that  $n^{n-1} - 1$  is divisible by  $(n - 1)^2$  for every positive integer  $n$ .

**Problem 4.** How many ordered triples  $(x, y, z)$  of integers are there such that  $|x| + |y| + |z| \leq 1000$ ?