

## Problems, March 2007

**Problem 1.** Show how to divide any triangle  $\mathcal{T}$  into 2007 triangles similar to  $\mathcal{T}$ .

**Problem 2.** Find all  $x$  that satisfy the inequality

$$\left(\frac{\sqrt{x+1}-1}{x}\right)^2 > x+1.$$

**Problem 3.** Download speed varies. Your browser posts a running estimate of the time needed to finish the download job. It does so by assuming that the *average* download speed so far will continue. The browser was downloading a large file. After 2 minutes, it estimated that there were 30 seconds left. For the next 5 minutes, the estimated remaining time to finish the job stayed at 30 seconds. What fraction of the file had been downloaded after these 5 minutes? (Assume that the program continued to function normally.)

**Problem 4.** Find the maximum value taken on by  $3x + 4y$  as  $(x, y)$  ranges over all ordered pairs such that  $x^2 + y^2 \leq 1$ .

**Problem 5.** The number  $2^{10}$  is “nearly”  $10^3$ , in the sense that  $2^{10}/10^3$  is close to 1. Show that there are positive integers  $a$  and  $b$  such that

$$0.998 < \frac{2^a}{10^b} < 1.002.$$

(Logarithms may be useful but are not essential.)