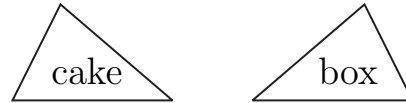


## Problems, March 2006

**Problem 1.** Let  $a_n = 1! + 2! + 3! + \cdots + n!$ . For what positive integers  $n$  is  $a_n$  a perfect square? Hint: Calculate for a while.

**Problem 2.** Alphonse and Beth ran one lap on the school 400 meter track. They started at the same time and place but ran in opposite directions, each at constant speed. Beth finished her lap 27.2 seconds after they passed each other, and Alphonse finished 42.5 seconds after they passed. How long did Beth take to run the lap? (A solution with some symmetry would be nice. There is even a geometric solution.)

**Problem 3.** A mathematician moonlighting as a pastry chef has made a triangular cake, frosted on top but not on the sides. The box for the cake fits perfectly, but only if the cake is put in upside down. Show how to cut the cake into some pieces so that the cake can be put in the box right side up. Try to find more than one solution.



**Problem 4.** (a) Show that if  $\triangle ABC$  is equilateral, then for any point  $P$  in the interior of  $\triangle ABC$ , the line segments  $PA$ ,  $PB$ , and  $PC$  can be rearranged to form a triangle. (b) Show that if  $\triangle ABC$  is not equilateral, there is a point  $P$  in the interior of the triangle such that the line segments  $PA$ ,  $PB$ , and  $PC$  can not be rearranged to form a triangle.

**Problem 5.** In how many ways can we express 90000 as a sum of consecutive odd integers?