

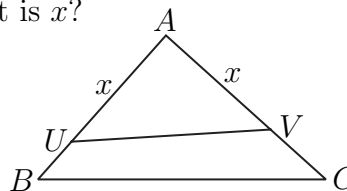
## Problems, February 2006

**Problem 1.** What are the first (leftmost) three digits in the decimal expansion of  $2^{2006}$ ? (Only a basic scientific calculator should be used.)

**Problem 2.** On March 21, Alfonso and Beti did a science experiment on the coast of Ecuador. Alfonso was standing 50 meters above sea level, and Beti was on a cliff 50 meters above Alfonso.

The sun was setting due west over the Pacific. At the moment Alfonso saw the sun dip below the horizon, he signalled Beti, who found with a stopwatch that the sun dipped below *her* horizon 22.5 seconds after it dipped below Alfonso's. Find the radius of the Earth. Hint: December 2005.

**Problem 3.** In  $\triangle ABC$ ,  $AB = 8$ ,  $AC = 9$ , and  $BC = 12$ . Points  $U$  and  $V$  are chosen on segments  $AB$  and  $AC$  with  $AU = AV = x$ . Suppose that  $UV$  cuts  $\triangle ABC$  into two parts of equal area. What is  $x$ ?



**Problem 4.** Let  $\mathcal{P}$  be a convex polygon with  $n$  sides. How many acute angles can  $\mathcal{P}$  have?

**Problem 5.** A calculator is defective: the only operation keys that work are the “-” key and the “ $1/x$ ” key. Show how to use this calculator to find (a) the square of a number; (b) the product of two numbers. (It is OK if your procedures do not work for very simple numbers such as 0 or 1.)

**Problem 6.** Alicia has 4 dollars and Beth has 2 dollars. A neutral third party tosses a fair coin. If the result is “head,” Beth gives Alicia 1 dollar, and if the result is “tail,” Alicia gives Beth 1 dollar. The coin tossing continues until one of the two players is bankrupt.

- What is the probability that it is Beth who ultimately goes bankrupt?
- What about if at the start Alicia has \$40 and Beth has \$20?