

Problems, November 2006

Problem 1. A six-sided convex polygon is inscribed in a circle. All its angles are equal. Show that its sides need not be equal. What can be said about seven-sided equal-angled inscribed convex polygons? Generalize.

Problem 2. Find an *exact* expression for the smallest positive real number x such that $\cos 3x + \sin 2x = 0$.

Problem 3. Suppose that $f(7) = 10^5$, and that $f(n+2) = f(n+1) + f(n)$ for every non-negative integer n . How many ordered pairs (a, b) of positive integers are there such that $f(0) = a$ and $f(1) = b$?

Problem 4. We have 99 pennies on a table, all of them with heads facing up.
(a) An *allowed move* consists of turning any 7 pennies upside down. Can we get all the pennies with heads facing down using only allowed moves?
(b) What about if an allowed move consists of turning any 8 pennies upside down?

Problem 5. Call a word w over the alphabet \mathcal{A} *odd* if each letter of \mathcal{A} occurs an odd number of times in w . Find a simple expression for the number of 101-letter odd words over the alphabet $\{a, b, c\}$.